

Intercept-slope model

Parametrization

The intercept-slope model is a convenient re-implementation of a commonly used construct, where

$$(a, b)$$

is bi-variate Gaussian with a Wishart prior for the precision matrix¹, and various forms of

$$\gamma(a + bz), \tag{1}$$

where z is a covariate and γ is a (random) scaling, goes into the linear predictor. Replicates of (a, b) is indexed by *subject*, $i = 1, \dots, n$, and the various scaling of Eq. 1 by *strata* $j = 1, \dots, m$, leading to a model for (a subset of)

$$\{\gamma_j(a_i + b_i z_{ij}), \quad i = 1, \dots, n, \quad j = 1, \dots, m\},$$

as not all combinations need to be present.

Hyperparameters

The hyperparameters are $(\theta_1, \theta_2, \theta_3)$ as in the model “iid2d” (related to the precisions of a and b , and their correlation), and $\theta_4 = \gamma_1, \dots, \gamma_{53} = \gamma_{50}$. Since m is defined in the input, only $\gamma_1, \dots, \gamma_m$ are used. m is limited to $m \leq 50$. **Please note** that all γ_i ’s are by default **fixed** to 1.

Specification

The is specified as

```
f(idx, model="intslope", hyper = ...,
  precision = exp(14),
  args.intslope = list(subject=i, strata=j, covariate = z))
```

The definition of the model is through the `args.intslope` argument, where `i` and `j` are factors/integers and `z` is numerical, all with same length N , say. The argument `idx`, index which row that is used for the linear predictor, hence values of `idx` must take integer values in the interval 1 to N . The precision argument, defines the tiny small noise added to each $\gamma(a + bz)$ to avoid a singular joint model. The `subject` and `strata` argument, is converted internally into integers 1, 2, ..., using

```
subject = as.numerical(as.factor(subject))
strata   = as.numerical(as.factor(strata))
```

and the results is shown after this conversion.

Hyperparameter specification and default values

`doc` Intecept-slope model with Wishart-prior

`hyper`

```
  theta1
    hyperid 16101
```

¹The documentation for the model “iid2d” gives the details of the definition of the parameterization of the precision matrix and the Wishart-prior.

```

    name log precision1
    short.name prec1
    initial 4
    fixed FALSE
    prior wishart2d
    param 4 1 1 0
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 16102
    name log precision2
    short.name prec2
    initial 4
    fixed FALSE
    prior none
    param
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta3
    hyperid 16103
    name logit correlation
    short.name cor
    initial 4
    fixed FALSE
    prior none
    param
    to.theta function(x) log((1 + x) / (1 - x))
    from.theta function(x) 2 * exp(x) / (1 + exp(x)) - 1
theta4
    hyperid 16104
    name gamma1
    short.name g1
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 16105
    name gamma2
    short.name g2
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta6

```

```

hyperid 16106
name gamma3
short.name g3
initial 1
fixed TRUE
prior normal
param 1 36
to.theta function(x) x
from.theta function(x) x
theta7
hyperid 16107
name gamma4
short.name g4
initial 1
fixed TRUE
prior normal
param 1 36
to.theta function(x) x
from.theta function(x) x
theta8
hyperid 16108
name gamma5
short.name g5
initial 1
fixed TRUE
prior normal
param 1 36
to.theta function(x) x
from.theta function(x) x
theta9
hyperid 16109
name gamma6
short.name g6
initial 1
fixed TRUE
prior normal
param 1 36
to.theta function(x) x
from.theta function(x) x
theta10
hyperid 16110
name gamma7
short.name g7
initial 1
fixed TRUE
prior normal
param 1 36
to.theta function(x) x
from.theta function(x) x

```

```

theta11
  hyperid 16111
  name gamma8
  short.name g8
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta12
  hyperid 16112
  name gamma9
  short.name g9
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta13
  hyperid 16113
  name gamma10
  short.name g10
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta14
  hyperid 16114
  name gamma11
  short.name g11
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta15
  hyperid 16115
  name gamma12
  short.name g12
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x

```

```

    from.theta function(x) x
theta16
  hyperid 16116
  name gamma13
  short.name g13
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta17
  hyperid 16117
  name gamma14
  short.name g14
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta18
  hyperid 16118
  name gamma15
  short.name g15
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta19
  hyperid 16119
  name gamma16
  short.name g16
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta20
  hyperid 16120
  name gamma17
  short.name g17
  initial 1
  fixed TRUE
  prior normal
  param 1 36

```

```

    to.theta function(x) x
    from.theta function(x) x
theta21
  hyperid 16121
  name gamma18
  short.name g18
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta22
  hyperid 16122
  name gamma19
  short.name g19
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta23
  hyperid 16123
  name gamma20
  short.name g20
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta24
  hyperid 16124
  name gamma21
  short.name g21
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta25
  hyperid 16125
  name gamma22
  short.name g22
  initial 1
  fixed TRUE
  prior normal

```

```

    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta26
    hyperid 16126
    name gamma23
    short.name g23
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta27
    hyperid 16127
    name gamma24
    short.name g24
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta28
    hyperid 16128
    name gamma25
    short.name g25
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta29
    hyperid 16129
    name gamma26
    short.name g26
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta30
    hyperid 16130
    name gamma27
    short.name g27
    initial 1
    fixed TRUE

```

```

    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 16131
    name gamma28
    short.name g28
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 16132
    name gamma29
    short.name g29
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 16133
    name gamma30
    short.name g30
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta34
    hyperid 16134
    name gamma31
    short.name g31
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta35
    hyperid 16135
    name gamma32
    short.name g32
    initial 1

```



```

    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta36
    hyperid 16136
    name gamma33
    short.name g33
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta37
    hyperid 16137
    name gamma34
    short.name g34
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta38
    hyperid 16138
    name gamma35
    short.name g35
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta39
    hyperid 16139
    name gamma36
    short.name g36
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta40
    hyperid 16140
    name gamma37
    short.name g37

```

```

    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta41
    hyperid 16141
    name gamma38
    short.name g38
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 16142
    name gamma39
    short.name g39
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta43
    hyperid 16143
    name gamma40
    short.name g40
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 16144
    name gamma41
    short.name g41
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 16145
    name gamma42

```

```

    short.name g42
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta46
    hyperid 16146
    name gamma43
    short.name g43
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta47
    hyperid 16147
    name gamma44
    short.name g44
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta48
    hyperid 16148
    name gamma45
    short.name g45
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta49
    hyperid 16149
    name gamma46
    short.name g46
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta50
    hyperid 16150

```

```

    name gamma47
    short.name g47
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 16151
    name gamma48
    short.name g48
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta52
    hyperid 16152
    name gamma49
    short.name g49
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta53
    hyperid 16153
    name gamma50
    short.name g50
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
constr FALSE
nrow.ncol FALSE
augmented FALSE
aug.factor 1
aug.constr
n.div.by
n.required FALSE
set.default.values TRUE
pdf intslope

```

Example

```
library(mvtnorm)
n = 300
idx = 1:n
nstrata = 3
strata = sample(1:nstrata, n, replace=TRUE)
nsubject = n %% nstrata
subject = sample(1:nsubject, n, replace=TRUE)
z = rnorm(n)
gam = c(1, 1 + rnorm(nstrata-1, sd = 0.2))

rho = sqrt(3)/2
Sigma = matrix(c(1/1, NA, NA, 1/2), 2, 2)
Sigma[1,2] = Sigma[2,1] = rho*sqrt(Sigma[1,1]*Sigma[2,2])

ab = rmvnorm(nsubject, sigma=Sigma)
a = ab[,1]
b = ab[,2]
s = 0.01
y = gam[strata] * (a[subject] + z * b[subject]) + rnorm(n, s = 0.01)
r = inla(y ~ -1 + f(idx, model = "intslope",
  args.intslope = list(subject = subject,
    strata = strata,
    covariates = z),
  ## this is for nstrata = 3
  hyper = list(gamma1 = list(fixed = TRUE),
    gamma2 = list(fixed = FALSE),
    gamma3 = list(fixed = FALSE))),
  data = list(y = y,
    idx = idx,
    subject = subject,
    strata = strata,
    z = z),
  control.family = list(hyper = list(
    prec = list(initial = log(1/s^2),
      fixed=TRUE))))))

summary(r)
```

Notes

- With $n_s = \max(\text{subject})$, the internal storage of this model is

$$(\gamma_{j_1}(a_{i_1} + z_1 b_{i_1}), \dots, \gamma_{j_N}(a_{i_N} + z_N b_{i_N}), a_1, \dots, a_{n_s}, b_1, \dots, b_{n_s}),$$

i.e. a vector of length $N + 2n_s$.