

Correlated random effects: iidkd

This model is available for dimensions $k = 2$, to 10. We describe in detail the case for $k = 3$ as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a different and more efficient parameterisation.

Parametrization

The $(k = 3)$ -dimensional Normal-Wishard model is used if one want to define three vectors of “random effects”, u and v and w , say, for which (u_i, v_i, w_i) are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

where the covariance matrix \mathbf{W}^{-1} is parameterised as $\mathbf{W} = \mathbf{L}\mathbf{L}^T$, where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \quad (1)$$

and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ can take any value. The number of hyperparameters are $k(k+1)/2$, which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$.

For these models the precision matrix \mathbf{W} is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(k+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > k + 1$$

and

$$c = 2^{(rk)/2} |\mathbf{R}|^{-r/2} \pi^{(k(k-1))/4} \prod_{j=1}^k \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (k+1)).$$

Hyperparameters

The hyperparameters are $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$.

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters $\{\theta_i\}$, and the conversion into interpretable quantities can be done using simulation as described below.

The prior for θ is **fixed** to be `wishartkd`, and number of prior parameters required are $1 + k(k+1)/2$. By default the prior-parameters are

$$(r = 100, \underbrace{1, \dots, 1}_{k \text{ times}}, 0, \dots, 0)$$

Specification

The model `iidkd` is specified as

```
y ~ f(i, model="iidkd", order=3, n = <length>) + ...
```

where $\text{order} = k = 3$, and the `iidkd` model is represented internally as one vector of length n ,

$$(u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_m, w_1, w_2, \dots, w_m)$$

where $n = 3m$, and n is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as 3 sum-to-zero constraints

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

Hyperparameter spesification and default values

(**Note:** The value “1048576” is just a code for “replace this by the default value”. As the default value depends on `order`, the was the easy way out for the moment.)

doc Gaussian random effect in $\text{dim}=k$ with Wishart prior

hyper

theta1

hyperid 29101

name theta1

short.name theta1

initial 1048576

fixed FALSE

prior wishartkd

param 11 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576
1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576
1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576
1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576
1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576

to.theta function(x) x

from.theta function(x) x

theta2

hyperid 29102

name theta2

short.name theta2

initial 1048576

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta3

hyperid 29103

```

    name theta3
    short.name theta3
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 29104
    name theta4
    short.name theta4
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 29105
    name theta5
    short.name theta5
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 29106
    name theta6
    short.name theta6
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 29107
    name theta7
    short.name theta7
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 29108
    name theta8
    short.name theta8
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 29109
    name theta9
    short.name theta9
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 29110
    name theta10
    short.name theta10
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 29111
    name theta11
    short.name theta11
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta12
  hyperid 29112
  name theta12
  short.name theta12
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta13
  hyperid 29113
  name theta13
  short.name theta13
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta14
  hyperid 29114
  name theta14
  short.name theta14
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta15
  hyperid 29115
  name theta15
  short.name theta15
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta16
  hyperid 29116
  name theta16
  short.name theta16

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta17
    hyperid 29117
    name theta17
    short.name theta17
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta18
    hyperid 29118
    name theta18
    short.name theta18
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta19
    hyperid 29119
    name theta19
    short.name theta19
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta20
    hyperid 29120
    name theta20
    short.name theta20
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta21
  hyperid 29121
  name theta21
  short.name theta21
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta22
  hyperid 29122
  name theta22
  short.name theta22
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta23
  hyperid 29123
  name theta23
  short.name theta23
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta24
  hyperid 29124
  name theta24
  short.name theta24
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta25
  hyperid 29125

```

```

    name theta25
    short.name theta25
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta26
    hyperid 29126
    name theta26
    short.name theta26
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta27
    hyperid 29127
    name theta27
    short.name theta27
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta28
    hyperid 29128
    name theta28
    short.name theta28
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta29
    hyperid 29129
    name theta29
    short.name theta29
    initial 1048576
    fixed FALSE

```



```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta30
    hyperid 29130
    name theta30
    short.name theta30
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 29131
    name theta31
    short.name theta31
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 29132
    name theta32
    short.name theta32
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 29133
    name theta33
    short.name theta33
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

theta34

hyperid 29134
name theta34
short.name theta34
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x

theta35

hyperid 29135
name theta35
short.name theta35
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x

theta36

hyperid 29136
name theta36
short.name theta36
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x

theta37

hyperid 29137
name theta37
short.name theta37
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x

theta38

hyperid 29138
name theta38
short.name theta38

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta39
    hyperid 29139
    name theta39
    short.name theta39
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta40
    hyperid 29140
    name theta40
    short.name theta40
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta41
    hyperid 29141
    name theta41
    short.name theta41
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 29142
    name theta42
    short.name theta42
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta43
  hyperid 29143
  name theta43
  short.name theta43
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta44
  hyperid 29144
  name theta44
  short.name theta44
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta45
  hyperid 29145
  name theta45
  short.name theta45
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta46
  hyperid 29146
  name theta46
  short.name theta46
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta47
  hyperid 29147

```

```

    name theta47
    short.name theta47
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta48
    hyperid 29148
    name theta48
    short.name theta48
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta49
    hyperid 29149
    name theta49
    short.name theta49
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta50
    hyperid 29150
    name theta50
    short.name theta50
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 29151
    name theta51
    short.name theta51
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta52
    hyperid 29152
    name theta52
    short.name theta52
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta53
    hyperid 29153
    name theta53
    short.name theta53
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta54
    hyperid 29154
    name theta54
    short.name theta54
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta55
    hyperid 29155
    name theta55
    short.name theta55
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

constr FALSE
nrow.ncol FALSE
augmented TRUE
aug.factor 1
aug.constr 1 2 3 4 5 6 7 8 9 10
n.div.by -1
n.required TRUE
set.default.values TRUE
status experimental
pdf iidkd

```

Example

Just simulate some data and estimate the parameters back. This is for **order=4**.

```

library(mvtnorm)

n <- 300
m <- 4
N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
  for (j in 1:m) {
    if (i != j) {
      Sigma[i, j] <- rho^abs(i-j) * sqrt(Sigma[i, i] * Sigma[j, j])
    }
  }
}

y <- c()
yy <- rmvnorm(n, sigma = Sigma)
for(i in 1:m) {
  y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
  ## set parameters using 'theta1'.
  ## these are the default parameters.
  hyper = list(theta1 = list(
    param = c(100, rep(1, m), rep(0, m*(m-1)/2)))),
  data = data.frame(i = 1:N, y),
  ## fix precision as we have exact observations
  control.family = list(hyper = list(
    prec = list(initial = 15, fixed = TRUE))),
  verbose = FALSE)

## this is how the internal parameters are defined

```

```

L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))

## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
                           mle = c(diag(LL), LL[lower.tri(LL)]),
                           inla = r$mode$theta))

## this gives a list of sampled matrices (stdev's and correlations)
xx <- inla.iidkd.sample(10^4, r, "i")
## compute the mean
qq <- matrix(rowMeans(matrix(unlist(xx), nrow = m^2)), m, m)

iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(diag(qq), qq[lower.tri(qq)]),
                           true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))

```