

# Weibull

## Parametrisation

The Weibull distribution is (**variant=0**)

$$f(y) = \alpha y^{\alpha-1} \lambda \exp(-\lambda y^\alpha), \quad \alpha > 0, \quad \lambda > 0$$

and (**variant=1**)

$$f(y) = \alpha y^{\alpha-1} \lambda^\alpha \exp(-(\lambda y)^\alpha), \quad \alpha > 0, \quad \lambda > 0$$

where

$\alpha$ : shape parameter.

## Link-function

The parameter  $\lambda$  is linked to the linear predictor as:

$$\lambda = \exp(\eta)$$

## Hyperparameters

The  $\alpha$  parameter is represented as

$$\alpha = \exp(S\theta)$$

and the prior is defined on  $\theta$ . The constant  $S$  currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of  $\alpha$  can make a huge difference.

## Specification

- family = **weibull** for regression and family = **weibullsurv** for survival
- Required arguments:  $y$  (to be given using **inla.surv()** for survival models), and **variant=0** (default) or 1 to define the parameterisation.

## Hyperparameter spesification and default values

### weibull

**doc** The Weibull likelihood

**hyper**

**theta**

**hyperid** 79001

**name** log alpha

**short.name** alpha

**initial** -2

**fixed** FALSE

**prior** pc.alphaw

**param** 5

**to.theta** function(x, sc = 0.1) log(x) / sc

**from.theta** function(x, sc = 0.1) exp(sc \* x)

**survival** FALSE

**discrete** FALSE

**link** default log neglog quantile

**pdf** weibull

## **weibullsurv**

```
doc The Weibull likelihood (survival)
hyper
  theta
    hyperid 79101
    name log alpha
    short.name alpha
    initial -2
    fixed FALSE
    prior pc.alphaw
    param 5
    to.theta function(x, sc = 0.1) log(x) / sc
    from.theta function(x, sc = 0.1) exp(sc * x)
survival TRUE
discrete FALSE
link default log neglog quantile
pdf weibull
```

## **Example**

In the following example we estimate the parameters in a simulated case

```
n = 1000
alpha = 1.1
beta = 2.2
x = c(scale(runif(n)))
eta = 1+beta*x
lambda = exp(eta)

for(variant in 0:1) {
  y = rweibull(n,
               shape= alpha,
               scale= if (variant == 0)
                       lambda^(-1/alpha)
               else
                       1/lambda)

  print(paste("VARIANT=", variant))
  event = rep(1,n)
  data = list(y=y, event=event, x=x)

  formula=inla.surv(y,event)~ x
  r=inla(formula,
         family ="weibullsurv",
         data=data,
         control.family = list(list(variant = variant)))
  print("SURV")
  print(summary(r))
}
```

```

formula= y ~ x
r=inla(formula,
      family ="weibull",
      data=data,
      control.family = list(list(variant = variant)))
print("REGRESSION")
print(summary(r))
}

```

## Notes

- Weibullsurv model can be used for right censored, left censored, interval censored data. If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observations, `time = time / max(time)` or similar.