

# Random walk model of order 1 (RW1)

## Parametrization

The random walk model of order 1 (RW1) for the Gaussian vector  $\mathbf{x} = (x_1, \dots, x_n)$  is constructed assuming independent increments:

$$\Delta x_i = x_i - x_{i-1} \sim \mathcal{N}(0, \tau^{-1})$$

The density for  $\mathbf{x}$  is derived from its  $n - 1$  increments as

$$\begin{aligned}\pi(\mathbf{x}|\tau) &\propto \tau^{(n-1)/2} \exp \left\{ -\frac{\tau}{2} \sum (\Delta x_i)^2 \right\} \\ &= \tau^{(n-1)/2} \exp \left\{ -\frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \right\}\end{aligned}$$

where  $\mathbf{Q} = \tau \mathbf{R}$  and  $\mathbf{R}$  is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW1 model, in this case the graph is modified so that last node  $x_n$  is neighbour of  $x_{n-1}$  and  $x_1$ .

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta = \log \tau$$

and the prior is defined on  $\theta$ .

## Specification

The RW1 model is specified inside the `f()` function as

```
f(<whatever>, model="rw1", values=<values>, cyclic=<TRUE|FALSE>,  
    hyper = <hyper>, scale.model = FALSE)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See next example for an application.

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior specification much easier. Default is `FALSE` so that the model is not scaled.

## Current recommended prior

If you do not know which prior to use, the current recommendation is

```
u = 1  
f(<whatever>, model="rw1", scale.model = TRUE  
    hyper = list(theta = list(prior="pc.prec", param=c(u,0.01))))  
inla.doc("pc.prec")
```

where `u` should be set to a value appropriate for your case:

**Gaussian likelihood (no link)** Set `u` to be the empirical standard deviation of your data

**Poisson likelihood and log link** Set `u` to 1

**Binomial and logit link** Set `u` to 0.5

**Binomial and probit link** Set `u` to 0.33

Increasing `u` gives a weaker prior, decreasing `u` gives a stronger prior.

## Hyperparameter specification and default values

**doc** Random walk of order 1

**hyper**

**theta**

**hyperid** 4001

**name** log precision

**short.name** prec

**prior** loggamma

**param** 1 5e-05

**initial** 4

**fixed** FALSE

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

**constr** TRUE

**nrow.ncol** FALSE

**augmented** FALSE

**aug.factor** 1

**aug.constr**

**n.div.by**

**n.required** FALSE

**set.default.values** FALSE

**min.diff** 1e-05

**pdf** rw1

## Example

```
n=100
```

```
z=seq(0,6,length.out=n)
```

```
y=sin(z)+rnorm(n,mean=0,sd=0.5)
```

```
data=data.frame(y=y,z=z)
```

```
formula=y~f(z,model="rw1",
```

```
      hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
```

```
result=inla(formula,data=data,family="gaussian")
```

## Notes

- The RW1 is intrinsic with rank deficiency 1.
- The RW1 model for irregular locations are supported although not described here.
- The term  $\frac{1}{2} \log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.