

Gaussian

Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s\tau(y-\mu)^2\right)$$

for continuously responses y where

μ : is the the mean

τ : is the precision

s : is a fixed scaling, $s > 0$.

Link-function

The mean and variance of y are given as

$$\mu \quad \text{and} \quad \sigma^2 = \frac{1}{s\tau}$$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The default behaviour is to represent the precision $\tau = \kappa_1$ where

$$\theta_1 = \log \kappa_1$$

and the prior is defined on θ_1 .

The more general formulation have a second (fixed) hyperparameter θ_2 which determines a fixed offset $1/\kappa_2$, $\theta_2 = \log \kappa_2$, for the variance (scaling not included) of the response. In this case,

$$1/\tau = 1/\kappa_1 + 1/\kappa_2$$

or

$$\tau = \frac{1}{1/\kappa_1 + 1/\kappa_2}$$

In the case where $1/\kappa_2$ is zero, then $\tau = \kappa_1$ and we are back to the default behaviour. We use the convension that $1/\kappa_2$ is zero if $1/\kappa_2 < \text{Machine}\$double.eps$, which is $\theta_2 \geq 36.05$ for common machines.

Specification

- family = `gaussian`
- Required arguments: y and s (argument `scale`)

The scalings have default value 1.

Hyperparameter specification and default values

doc The Gaussian likelihood

hyper

theta1

hyperid 65001
name log precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta function(x) log(x)
from.theta function(x) exp(x)

theta2

hyperid 65002
name log precision offset
short.name precoffset
initial 72.0873067782343
fixed TRUE
prior none
param
to.theta function(x) log(x)
from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default identity logit loga cauchit log logoffset

pdf gaussian

Example

The first example estimate the parameters in a simulated example with Gaussian responses, giving τ a Gamma-prior with parameters (1, 0.01) and initial value (for the optimisations) of $\exp(2.0)$. The second example shows the use of an fixed offset in the variance.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))
```

```

data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
              control.family = list(hyper = list(
                                    prec = list(
                                      prior = "loggamma",
                                      param = c(1.0,0.01),
                                      initial = 2))),
                                scale=scale, keep=TRUE)
summary(result)

## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
          data = data.frame(y, x),
          control.family = list(
            hyper = list(precoffset = list(initial = log(1/var0)))),
          verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "l", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")

```

Notes

An error is given if θ_2 is not fixed.