

The Berkson Measurement Error (MEB) model

Parametrization

This is an implementation of the Berkson measurement error model for a fixed effect. The observed covariate is w but it is x that goes into the linear predictor

$$\eta = \dots + \beta x + \dots ,$$

where $x = w + u$. The error term u is Gaussian with prior $\mathcal{N}(0, \tau_u \mathbf{D})^1$, where τ_u is the observational precision of the error $\text{Prec}(u|x)$ with possible heteroscedasticity, encoded in the entries d_i of the diagonal matrix \mathbf{D} . The vector s contains the fixed scalings $s = (d_1, \dots, d_n)$ (with n the number of data points).

Hyperparameters

This model has 2 hyperparameters, $\theta = (\theta_1, \theta_2)$. The hyperparameter specification is as follows:

$$\theta_1 = \beta$$

and the prior is defined on θ_1 ,

$$\theta_2 = \log(\tau_u)$$

and the prior is defined on θ_2 .

Specification

The MEB is specified inside the `f()` function as

```
f(w, [<weights>], model="meb", scale = <s>, values= <w>, hyper = <hyper>)
```

Here, `w` are the observed covariates, and the fixed scaling of the observational precision is given in argument `scale`. If the argument `scale` is not given, then s is set to 1.

Note that only the unique values of `w` are used, so if two or more elements of `w` are *identical*, then they refer to the *same* element in the covariate x . If data points with identical w values belong to different x values (e.g., different individuals), please add a *tiny* random value to w to make this difference obvious to the model.

Hyperparameter specification and default values

```
doc Berkson measurement error model
```

```
hyper
```

```
  theta1
```

```
    hyperid 3001
```

```
    name beta
```

```
    short.name b
```

```
    prior gaussian
```

```
    param 1 0.001
```

```
    initial 1
```

```
    fixed FALSE
```

```
    to.theta function(x) x
```

¹Note: The second argument in $\mathcal{N}(,)$ is the precision not the variance.

```

    from.theta function(x) x
  theta2
    hyperid 3002
    name prec.u
    short.name prec
    prior loggamma
    param 1 1e-04
    initial 6.90775527898214
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf meb

```

Example

```

n = 100
beta = 2
w = rnorm(n)
prec.u = 1
prec.y = 1
## heteroscedastic scaling
s = runif(n,min=0,max=1)
## true but unobserved covariate
x = w + rnorm(n, sd = 1/sqrt(s*prec.u))
y = 1 + beta*x + rnorm(n, sd = 1/sqrt(prec.y))

## prior parameters
prior.beta = c(0, 0.0001)
prior.prec.u = c(10, 9/prec.u)
prior.prec.y = c(10, 9/prec.y)

formula = y ~ f(w, model="meb", scale=s, values=w,
  hyper = list(
    beta = list(
      prior = "gaussian",

```

```

        param = prior.beta,
        fixed = FALSE
    ),
    prec.u = list(
        prior = "loggamma",
        param = prior.prec.u,
        initial = log(prec.u),
        fixed = FALSE
    )
)

r = inla(formula, data = data.frame(y, w, s),
        family = "gaussian",
        control.family = list(
            hyper = list(
                prec = list(param = prior.prec.y,
                            fixed = FALSE
                        )
            )
        )
)

```

Notes

- INLA provides the posteriors of $\nu_i = \beta x_i$ and NOT x_i .
- The posteriors of ν_i come (default) in the order given by the sorted (from low to high) values of **w**. The entry **\$ID** gives the mapping.
- The option **scale** defines the scaling in the same order as argument **values**. It is therefore advised to also give argument **values** when **scale** is used to be sure that they are consistent.