

## Proper random walk model of order 2 (PRW2)

### Parametrization

This is the proper version of the RW2 prior, defined for regular locations  $1, 2, \dots$ ,

$$x_t = 2\gamma x_{t-1} - \gamma^2 x_{t-2} + \epsilon_t, \quad |\gamma| < 1$$

where  $\epsilon_t$  is iid zero mean Gaussian noise. This process is made stationary with marginal precision  $\kappa$ , and range  $r$ , which is the 'distance' or 'time' to small correlation, is defined (implicitly) as

$$\rho = \exp\left(-\frac{\sqrt{12}h}{r}\right)$$

where  $h$  is (typical) distance or time, 'step-size', between the locations where the process is defined (here  $h = 1$ ). The correlation function of the process is found to be

$$\rho(h) = \left(1 + h \frac{1 - \gamma^2}{1 + \gamma^2}\right) \gamma^h, \quad h = 0, 1, 2, \dots$$

Similar to RW1 and RW2, this model allows for irregular locations (`loc`) and the process is then derived from the corresponding limiting process in continuous time.

Note that  $r$  and  $h$  is in real scale, ie the scale of `loc`.

### Hyperparameters

This model has two hyperparameters, the marginal precision  $\kappa$  and the range  $r$ . These are defined as

$$\theta_1 = \log(\kappa)$$

$$\theta_2 = \log(r)$$

and the priors are given for  $(\theta_1, \theta_2)$ .

### Specification

This model is given as `f(loc, model="prw2", ...)`.

Note that `constr=FALSE` by default for this model, unlike the similar but intrinsic, RW2 model.

The default is use PC-priors for both parameters, see `inla.doc("pc.prec")` and `inla.doc("pc.prw2.range")`. The prior `pc.prw2.range` is currently fixed and not possible to replace, as this prior is viral for the PRW2 model construction and motivation itself.

### Hyperparameter spesification and default values

`doc` Proper random walk of order 2

`hyper`

`theta1`

`hyperid 6103`

`name log precision`

`short.name prec`

`prior pc.prec`

`param 1 0.01`

`initial 4`

```

    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
  theta2
    hyperid 6102
    name log range
    short.name range
    prior pc.prw2.range
    param 0 0 0 0
    initial 3
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

  constr FALSE

  nrow.ncol FALSE

  augmented FALSE

  aug.factor 1

  aug.constr

  n.div.by

  n.required FALSE

  set.default.values FALSE

  pdf prw2

```

## Example

```

n <- 200
loc <- seq(0, n-1, length.out=n)
h.size <- diff(range(loc)) / (n-1)
f.true <- (sin(2*pi*(loc/n)^2))^3
s = 0.2
y <- f.true + rnorm(n, sd = s)

plot(loc, y, pch = 19)
lines(loc, f.true, type='l', lwd = 3, col='blue')

r <- inla(y ~ -1 + f(loc, model = "prw2", values = loc,
  hyper = list(
    range = list(
      param = c(50, 0.5, h.size, 0))),
  family = "normal",
  control.family = list(
    hyper = list(prec = list(initial = log(1/s^2),
      fixed = TRUE))),
  data = data.frame(y, loc))
lines(loc, r$summary.linear.predictor$mean, lwd = 3, col = "red")

```

```

## we can now change the resolution and get the 'same' results
nn <- 2*n
lloc <- seq(0, n-1, length.out = nn)
yy <- numeric(nn)
for(i in 1:n) {
  yy[1 + (i-1) * 2] <- y[i]
  yy[2 + (i-1) * 2] <- NA
}
hh.size <- diff(range(lloc)) / (nn-1)

rr <- inla(yy ~ -1 + f(lloc, model = "prw2", values = lloc,
  hyper = list(
    ## note that '50' is the same as its in the real scale
    range = list(param = c(50, 0.5, hh.size, 0))),
  family = "normal",
  control.family = list(hyper = list(prec = list(initial = log(1/s^2),
    fixed = TRUE))),
  data = data.frame(yy, lloc))
lines(lloc, rr$summary.linear.predictor$mean, lwd = 3, col = "red")

## once more
nnn <- 4*n
llloc <- seq(0, n-1, length.out = nnn)
yyy <- numeric(nnn)
for(i in 1:n) {
  yyy[1 + (i-1) * 4] <- y[i]
  yyy[2:4 + (i-1) * 4] <- NA
}
hhh.size <- diff(range(llloc)) / (nnn-1)

rrr <- inla(yyy ~ -1 + f(llloc, model = "prw2", values = llloc,
  hyper = list(
    ## again, '50' is not changed
    range = list(param = c(50, 0.5, hhh.size, 0))),
  family = "normal",
  control.family = list(hyper = list(prec = list(initial = log(1/s^2),
    fixed = TRUE))),
  data = data.frame(yy, lloc))
lines(llloc, rrr$summary.linear.predictor$mean, lwd = 3, col = "red")

inla.dev.new()
res <- cbind(r$summary.linear.predictor$mean,
  rr$summary.linear.predictor$mean[seq(1, nn, by = 2)],
  rrr$summary.linear.predictor$mean[seq(1, nnn, by = 4)])
pairs(res)

inla.dev.new()
plot(r$marginals.hyperpar[[2]], pch = 19)
lines(rr$marginals.hyperpar[[2]], lwd = 3, col = "blue")
lines(rrr$marginals.hyperpar[[2]], lwd = 3, col = "red")

```

## Notes

The details for this model is available in the PhD thesis with link [HERE](#) (ADD LINK LATER).