

# New 0inflated models: Poisson & Binomial

## Parametrisation

This is a new implementation (Nov'22) of zero-inflated Poisson and Binomial likelihood, where we will allow for a linear predictor in both the zero-inflation and in the mean, but one of them needs to consist of fixed effects only. This means the setup will be somewhat different than for other likelihood models.

## Details

The zero-inflated likelihood  $f_0(y|\dots)$  is defined as

$$f_0(y|\eta_1, \eta_2) = p(\eta_1)1_{[y=0]} + (1 - p(\eta_1))f(y|\eta_2)$$

where  $f(y|\dots)$  is either Poisson or Binomial. We allow for two linear predictors in the model, but one needs to be “simple” (i.e. only consists of fixed effects). The other is general and defined via the formula. Normally, the zero-inflation probability is simpler (`family="0..."`)

$$\eta_1 = \text{simple} \quad \eta_2 = \text{formula}$$

but they can also be swapped (`family="0...S"`)

$$\eta_1 = \text{formula} \quad \eta_2 = \text{simple}$$

## Link-function

This is similar to Poisson and Binomial.

The link-function for the 'simple'-model must be given by argument `link.simple` in the `control.family`-argument. Only link-models without covariates/parameters are currently available. The examples later on show how this is done.

## Hyperparameters

All parameters in the simple model are treated as hyperparameters. The  $j$ 'th element of  $\eta_1$  is

$$(\eta_1)_j = \sum_{i=1}^m m\beta_i x_{ij}$$

for covariates  $x_1, \dots$ , where  $m$  is maximum 10. An intercept in this model has to be defined manually by adding a constant covariate vector.

## Specification

- `family = 0poisson`
- `family = 0poissonS`
- `family = 0binomial`
- `family = 0binomialS`
- Required arguments: As for the Poisson and Binomial (but how these arguments are given, will differ). Optional argument `link.simple`.

## Hyperparameter spesification and default values

### Opoisson

**doc** New 0-inflated Poisson

**hyper**

**theta1**

**hyperid** 56201  
**name** beta1  
**short.name** beta1  
**initial** -4  
**fixed** FALSE  
**prior** normal  
**param** -4 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta2**

**hyperid** 56202  
**name** beta2  
**short.name** beta2  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta3**

**hyperid** 56203  
**name** beta3  
**short.name** beta3  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta4**

**hyperid** 56204  
**name** beta4  
**short.name** beta4  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta5**

**hyperid** 56205  
**name** beta5

```

    short.name beta5
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56206
    name beta6
    short.name beta6
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56207
    name beta7
    short.name beta7
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56208
    name beta8
    short.name beta8
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56209
    name beta9
    short.name beta9
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 56210

```

```

    name beta10
    short.name beta10
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x

status experimental
survival FALSE
discrete TRUE
link default log quantile
link.simple default logit cauchit probit cloglog
pdf 0inflated

```

### **0poissonS**

**doc** New 0-inflated Poisson Swap

**hyper**

#### **theta1**

```

    hyperid 56301
    name beta1
    short.name beta1
    initial -4
    fixed FALSE
    prior normal
    param -4 10
    to.theta function(x) x
    from.theta function(x) x

```

#### **theta2**

```

    hyperid 56302
    name beta2
    short.name beta2
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x

```

#### **theta3**

```

    hyperid 56303
    name beta3
    short.name beta3
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x

```

```

    from.theta function(x) x
theta4
    hyperid 56304
    name beta4
    short.name beta4
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 56305
    name beta5
    short.name beta5
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56306
    name beta6
    short.name beta6
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56307
    name beta7
    short.name beta7
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56308
    name beta8
    short.name beta8
    initial 0
    fixed FALSE
    prior normal
    param 0 10

```

```

    to.theta function(x) x
    from.theta function(x) x
theta9
  hyperid 56309
  name beta9
  short.name beta9
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 56310
  name beta10
  short.name beta10
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
status experimental
survival FALSE
discrete TRUE
link default logit loga cauchit probit cloglog loglog log sslogit logitoffset quantile pquantile robit sn powerlogit
link.simple default log
pdf 0inflated

Obinomial
doc New 0-inflated Binomial
hyper
  theta1
    hyperid 56401
    name beta1
    short.name beta1
    initial -4
    fixed FALSE
    prior normal
    param -4 10
    to.theta function(x) x
    from.theta function(x) x
  theta2
    hyperid 56402
    name beta2
    short.name beta2

```

```

    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta3
    hyperid 56403
    name beta3
    short.name beta3
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 56404
    name beta4
    short.name beta4
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 56405
    name beta5
    short.name beta5
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56406
    name beta6
    short.name beta6
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56407
    name beta7

```

```

    short.name beta7
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56408
    name beta8
    short.name beta8
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56409
    name beta9
    short.name beta9
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 56410
    name beta10
    short.name beta10
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x

status experimental
survival FALSE
discrete TRUE
link default logit loga cauchit probit cloglog loglog log
link.simple default logit cauchit probit cloglog
pdf 0inflated

```

## **Opoisson**

**doc** New 0-inflated Binomial Swap

**hyper**

**theta1**

**hyperid** 56501  
**name** beta1  
**short.name** beta1  
**initial** -4  
**fixed** FALSE  
**prior** normal  
**param** -4 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta2**

**hyperid** 56502  
**name** beta2  
**short.name** beta2  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta3**

**hyperid** 56503  
**name** beta3  
**short.name** beta3  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta4**

**hyperid** 56504  
**name** beta4  
**short.name** beta4  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta5**

**hyperid** 56505  
**name** beta5  
**short.name** beta5  
**initial** 0

```

    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56506
    name beta6
    short.name beta6
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56507
    name beta7
    short.name beta7
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56508
    name beta8
    short.name beta8
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56509
    name beta9
    short.name beta9
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 56510
    name beta10
    short.name beta10

```

```
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x

status experimental
survival FALSE
discrete TRUE
link default logit loga cauchit probit cloglog loglog log
link.simple default logit cauchit probit cloglog
pdf 0inflated
```

## Example: Poisson

```
sim.poisson <- function(prob, m)
{
  stopifnot(length(prob) == length(m) && length(prob) > 0)
  n <- length(m)
  y <- numeric(n)
  event <- (runif(n) < prob)
  idx.zero <- which(event)
  idx.non.zero <- which(!event)
  y[idx.zero] <- 0
  y[idx.non.zero] <- rpois(length(idx.non.zero), lambda = m[idx.non.zero])
  return (y)
}

## chose link-function to use for the zero-inflation probability
link.simple <- "logit"
inv.link <- inla.link.invlogit
## link.simple <- "probit"
## inv.link <- inla.link.invprobit
## link.simple <- "cloglog"
## inv.link <- inla.link.invcloglog

n <- 1000
z <- rnorm(n, sd = 0.3)
x <- rnorm(n, sd = 0.2)
xx <- rnorm(n, sd = 0.3)
zz <- rnorm(n, sd = 0.2)
E <- runif(n, min = 0.8, max = 1/0.8)

beta <- c(1, 1.1, 2.1, 0, -2, 1.2, 2.2, 0)
eta2 <- beta[1] + beta[2] * xx + beta[3] * zz + beta[4] * xx * zz
eta1 <- beta[5] + beta[6] * x + beta[7] * z + beta[8] * x * z
prob <- inv.link(eta1)
m <- E*exp(eta2)

ok <- FALSE
while(!ok) {
  y <- sim.poisson(prob, m)
  ok <- !all(y == 0)
}

## head(data.frame(y, E, x, z, xx, zz))

r <- inla(
  inla.mdata(cbind(y, E), cbind(1, x, z, x*z)) ~ 1 + xx + zz + xx*zz,
  family = "0poisson",
  data = data.frame(y, E, x, z, xx, zz),
  control.fixed = list(prec = 1, prec.intercept = 1),
  control.compute = list(cpo = TRUE),
  control.family = list(link.simple = link.simple,
    hyper = list(beta1 = list(param = c(0, 1)),
      beta2 = list(param = c(0, 1)),
      beta3 = list(param = c(0, 1)),
      beta4 = list(param = c(0, 1)),
      beta5 = list(param = c(0, 1))))))

rr <- inla(
```

```

inla.mdata(cbind(y, E), cbind(1, xx, zz, xx*zz)) ~ 1 + x + z + x*z,
family = "0poissonS",
data = data.frame(y, E, x, z, xx, zz),
control.fixed = list(prec = 1, prec.intercept = 1),
control.compute = list(cpo = TRUE),
## in this case we need to define link.simple as the main link
control.family = list(control.link = list(model = link.simple),
                      hyper = list(beta1 = list(param = c(0, 1)),
                                   beta2 = list(param = c(0, 1)),
                                   beta3 = list(param = c(0, 1)),
                                   beta4 = list(param = c(0, 1)),
                                   beta5 = list(param = c(0, 1)))))

summary(r)
summary(rr)

res <- cbind("beta" = beta,
            "0poisson" = c(r$summary.fixed$mean, r$summary.hyperpar$mean),
            "0poissonS" = c(rr$summary.hyperpar$mean, rr$summary.fixed$mean))
res <- cbind(res,
            diff = (res[, 2]-beta),
            diffS = (res[, 3]-beta),
            "diff/sd" = (res[, 2]-beta) / c(r$summary.fixed$sd, r$summary.hyperpar$sd),
            "diffS/sd" = (res[, 3]-beta) / c(rr$summary.hyperpar$sd, rr$summary.fixed$sd))
mm <- nrow(res) %/% 2
rownames(res) <- c(paste0("beta", 1:mm, ".poisson"), paste0("beta", 1:mm, ".prob"))
print(round(dig = 2, res))

```

## Example: Binomial

```
sim.binomial <- function(prob, p, size)
{
  ## - prob=zero-inflation-prob
  ## - binomial(size, p)
  stopifnot(length(prob) == length(p) && length(prob) == length(size)
            && length(prob) > 0)
  n <- length(prob)
  y <- numeric(n)
  event <- (runif(n) < prob)
  idx.zero <- which(event)
  idx.non.zero <- which(!event)
  y[idx.zero] <- 0
  y[idx.non.zero] <- rbinom(length(idx.non.zero),
                           size = size[idx.non.zero],
                           prob = p[idx.non.zero])

  return (y)
}

n <- 1000
z <- rnorm(n, sd = 0.3)
x <- rnorm(n, sd = 0.2)
xx <- rnorm(n, sd = 0.3)
zz <- rnorm(n, sd = 0.2)
Ntrials <- sample(1:10, n, replace = TRUE)

## chose link-function to use for the zero-inflation probability
link.simple <- "logit"
inv.link <- inla.link.invlogit
## link.simple <- "probit"
## inv.link <- inla.link.invprobit
## link.simple <- "cloglog"
## inv.link <- inla.link.invcloglog

beta <- c(1, 1.1, 2.1, 0, -2, 1.2, 2.2, 0)
eta2 <- beta[1] + beta[2] * xx + beta[3] * zz + beta[4] * xx * zz
eta1 <- beta[5] + beta[6] * x + beta[7] * z + beta[8] * x * z
prob <- inv.link(eta1)
p <- 1/(1 + exp(-eta2))

ok <- FALSE
while(!ok) {
  y <- sim.binomial(prob, p, Ntrials)
  ok <- !all(y == 0)
}

## head(data.frame(y, Ntrials, x, z, xx, zz))

r <- inla(
  inla.mdata(cbind(y, Ntrials), cbind(1, x, z, x*z)) ~ 1 + xx + zz + xx*zz,
  family = "0binomial",
  data = data.frame(y, Ntrials, x, z, xx, zz),
  control.fixed = list(prec = 1, prec.intercept = 1),
  control.compute = list(cpo = TRUE),
  control.family = list(link.simple = link.simple,
                        hyper = list(beta1 = list(param = c(0, 1)),
                                      beta2 = list(param = c(0, 1)),
```

```

beta3 = list(param = c(0, 1)),
beta4 = list(param = c(0, 1)),
beta5 = list(param = c(0, 1))))

rr <- inla(
  inla.mdata(cbind(y, Ntrials), cbind(1, xx, zz, xx*zz)) ~ 1 + x + z + x*z,
  family = "0binomialS",
  data = data.frame(y, Ntrials, x, z, xx, zz),
  control.fixed = list(prec = 1, prec.intercept = 1),
  control.compute = list(cpo = TRUE),
  ## in this case we need to define link.simple as the main link
  control.family = list(control.link = list(model = link.simple),
    hyper = list(beta1 = list(param = c(0, 1)),
      beta2 = list(param = c(0, 1)),
      beta3 = list(param = c(0, 1)),
      beta4 = list(param = c(0, 1)),
      beta5 = list(param = c(0, 1)))))

summary(r)
summary(rr)

res <- cbind("beta" = beta,
  "0binomial" = c(r$summary.fixed$mean, r$summary.hyperpar$mean),
  "0binomialS" = c(rr$summary.hyperpar$mean, rr$summary.fixed$mean))
res <- cbind(res,
  diff = (res[, 2]-beta),
  diffS = (res[, 3]-beta),
  "diff/sd" = (res[, 2]-beta) / c(r$summary.fixed$sd, r$summary.hyperpar$sd),
  "diffS/sd" = (res[, 3]-beta) / c(rr$summary.hyperpar$sd, rr$summary.fixed$sd))
mm <- nrow(res) %/% 2
rownames(res) <- c(paste0("beta", 1:mm, ".binomial"), paste0("beta", 1:mm, ".prob"))
print(round(dig = 2, res))

```