

# Zero-inflated models: Poisson, Binomial, negative Binomial and BetaBinomial

## Parametrisation

There is support two types of zero-inflated models, which we name type 0 and type 1. These are defined for both the Binomial, the Poisson, the censored Poisson, the negative Binomial and BetaBinomial likelihood. For simplicity we will describe only the Poisson as the other cases are similar.

### Type 0

The (type 0) likelihood is defined as

$$\text{Prob}(y \mid \dots) = p \times 1_{[y=0]} + (1 - p) \times \text{Poisson}(y \mid y > 0)$$

where  $p$  is a hyperparameter where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and  $\theta$  is the internal representation of  $p$ ; meaning that the initial value and prior is given for  $\theta$ . This is model is called `zeroinflatedpoisson0` (and `zeroinflatedbinomial0`).

### Type 1

The (type 1) likelihood is defined as

$$\text{Prob}(y \mid \dots) = p \times 1_{[y=0]} + (1 - p) \times \text{Poisson}(y)$$

where  $p$  is a hyperparameter where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and  $\theta$  is the internal representation of  $p$ ; meaning that the initial value and prior is given for  $\theta$ . This is model is called `zeroinflatedpoisson1` (and `zeroinflatedbinomial1`).

## Link-function

As for the Poisson, the Binomial the negative Binomial and the BetaBinomial.

## Hyperparameters

For Poisson and the Binomial, there is one hyperparameter; where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior and initial value is is given for  $\theta$ .

For the negative Binomial and BetaBinomial, there are two hyperparameters. The overdispersion parameter  $n$  for the negative Binomial is represented as

$$\theta_1 = \log(n)$$

and the prior is defined on  $\theta_1$ . The zero-inflation parameter  $p$ , is represented as

$$p = \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior and initial value is is given for  $\theta_2$ . For the BetaBinomial it is similar.

## Specification

- `family="zeroinflatedbinomial0"`
- `family="zeroinflatedbinomial1"`
- `family="zeroinflatednbinomial0"`
- `family="zeroinflatednbinomial1"`
- `family="zeroinflatedpoisson0"`
- `family="zeroinflatedpoisson1"`
- `family="zeroinflatedcenpoisson0"`
- `family="zeroinflatedcenpoisson1"`
- `family="zeroinflatedbetabinomial0"`
- `family="zeroinflatedbetabinomial1"`
- Required arguments: As for the Binomial, the negative Binomial, BetaBinomial and Poisson likelihood.

## Hyperparameter specification and default values

### Zeroinflated Binomial Type 0

`doc` Zero-inflated Binomial, type 0

`hyper`

`theta`

`hyperid` 90001

`name` logit probability

`short.name` prob

`output.name` zero-probability parameter for zero-inflated binomial\_0

`output.name.intern` intern zero-probability parameter for zero-inflated binomial\_0

`initial` -1

`fixed` FALSE

`prior` gaussian

`param` -1 0.2

`to.theta` `function(x) log(x / (1 - x))`

`from.theta` `function(x) exp(x) / (1 + exp(x))`

`survival` FALSE

`discrete` FALSE

`link` default logit loga cauchit probit cloglog ccloglog loglog robit sn

`pdf` zeroinflated

## Zeroinflated Binomial Type 1

doc Zero-inflated Binomial, type 1

hyper

theta

hyperid 91001  
name logit probability  
short.name prob  
output.name zero-probability parameter for zero-inflated binomial\_1  
output.name.intern intern zero-probability parameter for zero-inflated binomial\_1  
initial -1  
fixed FALSE  
prior gaussian  
param -1 0.2  
to.theta function(x) log(x / (1 - x))  
from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

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## Zeroinflated NegBinomial Type 0

doc Zero inflated negBinomial, type 0

hyper

theta1

hyperid 95001  
name log size  
short.name size  
output.name size for nbinomial\_0 zero-inflated observations  
output.name.intern log size for nbinomial\_0 zero-inflated observations  
initial 2.30258509299405  
fixed FALSE  
prior pc.mgamma  
param 7  
to.theta function(x) log(x)  
from.theta function(x) exp(x)

theta2

hyperid 95002  
name logit probability  
short.name prob  
output.name zero-probability parameter for zero-inflated nbinomial\_0

```

    output.name.intern intern zero-probability parameter for zero-inflated nbinomial_0
    initial -1
    fixed FALSE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

Zeroinflated NegBinomial Type 1
doc Zero inflated negBinomial, type 1
hyper
  theta1
    hyperid 96001
    name log size
    short.name size
    output.name size for nbinomial_1 zero-inflated observations
    output.name.intern log size for nbinomial_1 zero-inflated observations
    initial 2.30258509299405
    fixed FALSE
    prior pc.mgamma
    param 7
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
  theta2
    hyperid 96002
    name logit probability
    short.name prob
    output.name zero-probability parameter for zero-inflated nbinomial_1
    output.name.intern intern zero-probability parameter for zero-inflated nbinomial_1
    initial -1
    fixed FALSE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

```

## Zeroinflated BetaBinomial Type 0

doc Zero-inflated Beta-Binomial, type 0

hyper

theta1

hyperid 88001  
name overdispersion  
short.name rho  
output.name rho for zero-inflated betabinomial\_0  
output.name.intern rho\_intern for zero-inflated betabinomial\_0  
initial 0  
fixed FALSE  
prior gaussian  
param 0 0.4  
to.theta function(x) log(x / (1 - x))  
from.theta function(x) exp(x) / (1 + exp(x))

theta2

hyperid 88002  
name logit probability  
short.name prob  
output.name zero-probability parameter for zero-inflated betabinomial\_0  
output.name.intern intern zero-probability parameter for zero-inflated betabinomial\_0  
initial -1  
fixed FALSE  
prior gaussian  
param -1 0.2  
to.theta function(x) log(x / (1 - x))  
from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete TRUE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

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## Zeroinflated BetaBinomial Type 1

doc Zero-inflated Beta-Binomial, type 1

hyper

theta1

hyperid 89001  
name overdispersion  
short.name rho  
output.name rho for zero-inflated betabinomial\_1

```

    output.name.intern rho_intern for zero-inflated betabinomial_1
    initial 0
    fixed FALSE
    prior gaussian
    param 0 0.4
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
theta2
    hyperid 89002
    name logit probability
    short.name prob
    output.name zero-probability parameter for zero-inflated betabinomial_1
    output.name.intern intern zero-probability parameter for zero-inflated betabinomial_
    initial -1
    fixed FALSE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete TRUE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

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Zeroinflated Poisson Type 0
doc Zero-inflated Poisson, type 0
hyper
    theta
        hyperid 85001
        name logit probability
        short.name prob
        output.name zero-probability parameter for zero-inflated poisson_0
        output.name.intern intern zero-probability parameter for zero-inflated poisson_0
        initial -1
        fixed FALSE
        prior gaussian
        param -1 0.2
        to.theta function(x) log(x / (1 - x))
        from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

```

## Zeroinflated Poisson Type 1

doc Zero-inflated Poisson, type 1

hyper

theta

hyperid 86001

name logit probability

short.name prob

output.name zero-probability parameter for zero-inflated poisson\_1

output.name.intern intern zero-probability parameter for zero-inflated poisson\_1

initial -1

fixed FALSE

prior gaussian

param -1 0.2

to.theta function(x) log(x / (1 - x))

from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

## Zeroinflated Censored Poisson Type 0

doc Zero-inflated censored Poisson, type 0

hyper

theta

hyperid 87101

name logit probability

short.name prob

output.name zero-probability parameter for zero-inflated poisson\_0

output.name.intern intern zero-probability parameter for zero-inflated poisson\_0

initial -1

fixed FALSE

prior gaussian

param -1 0.2

to.theta function(x) log(x / (1 - x))

from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

## Zeroinflated Censored Poisson Type 1

doc Zero-inflated censored Poisson, type 1

hyper

theta

hyperid 87201

name logit probability

short.name prob

output.name zero-probability parameter for zero-inflated poisson\_1

output.name.intern intern zero-probability parameter for zero-inflated poisson\_1

initial -1

fixed FALSE

prior gaussian

param -1 0.2

to.theta function(x) log(x / (1 - x))

from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

## Example

In the following example we estimate the parameters in a simulated example for both type 0 and type 1.

### Poisson

```
## type 0
```

```
n=100
```

```
a = 1
```

```
b = 1
```

```
z = rnorm(n)
```

```
eta = a + b*z
```

```
p = 0.2
```

```
E = sample(c(1,5,10,15), size=n, replace=TRUE)
```

```
lambda = E*exp(eta)
```

```
## first sample y|y>0
```

```
y = rpois(n, lambda = lambda)
```

```
is.zero = (y == 0)
```

```
while(sum(is.zero) > 0)
```

```
{
```

```
  y[is.zero] = rpois(sum(is.zero), lambda[is.zero])
```

```
  is.zero = (y == 0)
```

```
}
```



```
## then set some of these to zero
y[ rbinom(n, size=1, prob=p) == 1 ] = 0

data = list(y=y,z=z)
formula = y ~ 1+z
result0 = inla(formula, family = "zeroinflatedpoisson0", data = data, E=E)
summary(result0)

## type 1
y = rpois(n, lambda = lambda)
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result1 = inla(formula, family = "zeroinflatedpoisson1", data = data, E=E)
summary(result1)
```

## Binomial

```
## type 0
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
p = 0.2
Ntrials = sample(c(1,5,10,15), size=n, replace=TRUE)
prob = exp(eta)/(1 + exp(eta))

y = rbinom(n, size = Ntrials, prob = prob)
is.zero = (y == 0)
while(sum(is.zero) > 0)
{
  y[is.zero] = rbinom(sum(is.zero), size = Ntrials[is.zero], prob = prob[is.zero])
  is.zero = (y == 0)
}
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result0 = inla(formula, family = "zeroinflatedbinomial0", data = data, Ntrials = Ntrials)
summary(result0)

## type 1
y = rbinom(n, size = Ntrials, prob = prob)
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result1 = inla(formula, family = "zeroinflatedbinomial1", data = data, Ntrials=Ntrials)
summary(result1)
```

## Advanced example

In the following example we estimate the parameters in a simulated example for a type0 likelihood, where one linear predictor enters the zero-probability and one other linear predictor enters the non-zero Poisson for example. The same trick can be used for other models of type0. The trick is that the likelihood

$$p^* 1_{[y=0]} + (1 - p^*) P(y|y > 0)$$

can be reformulated as a Bernoulli likelihood for the “class”-variable

$$z = \begin{cases} 1, & \text{if } y = 0 \\ 0, & \text{if } y > 0. \end{cases}$$

where  $p^*$  is the probability for success, and zero-inflated type0 likelihood (with fixed  $p = 0$ ) for those  $y > 0$ . Since  $p^*$  and the linear predictor in  $P$  is separated into two likelihoods, we can apply one linear predictor to each one, hence extend the basic model to cases where  $p^*$  also depends on a linear predictor. Here is a small simulated example doing this.

```
require(INLA)

n = 100
a = 0.5
b = 1.5
x1 = rnorm(n, sd = 0.5)

eta.z = -a - b*x1
z = rbinom(n, 1, inla.link.logit(eta.z, inverse=TRUE))
n.y = sum(z)

x2 = rnorm(n.y, sd = 0.5)
eta.y = a + b*x2
lambda = exp(eta.y)
y = rpois(n.y, lambda)

is.zero = (y == 0)
while(sum(is.zero) > 0)
{
  y[is.zero] = rpois(sum(is.zero), lambda[is.zero])
  is.zero = (y == 0)
}

Y = matrix(NA, n + n.y, 2)
Y[1:n, 1] = z
Y[n + 1:n.y, 2] = y

form = Y ~ 0 + mu.z + mu.y + cov.z + cov.y
ldat = list(
  Y=Y,
  mu.z=rep(1:0, c(n, n.y)),
  mu.y=rep(0:1, c(n, n.y)),
  cov.z=c(x1, rep(NA,n.y)),
  cov.y=c(rep(NA, n), x2))
```

```

res <- inla(form, data=ldat,
            family=c('binomial', 'zeroinflatedpoisson0'),
            control.family=list(
                list(),
                list(hyper = list(
                    prob = list(
                        initial = -20,
                        fixed = TRUE))))))
round(res$summary.fix, 4)

```

## Notes

None.

## Extentions

There are some extentions available which currently is only implemented for the cases where its needed/requested.

**Type 2** Is like Type 1 but where (for the Poisson)

$$p = 1 - \left( \frac{E \exp(x)}{1 + E \exp(x)} \right)^\alpha$$

where  $\alpha > 0$  is the hyperparameter instead of  $p$  (and  $E \exp(x)$  is the mean). Available for Poisson as `zeroinflatedpoisson2`, for binomial as `zeroinflatedbinomial2` and for the negative binomial as `zeroinflatednbinomial2`.

The internal representation is  $\theta = \log(\alpha)$  and prior is defined on  $\log(\alpha)$ .

## Zeroinflated Poisson Type 2

**doc** Zero-inflated Poisson, type 2

**hyper**

**theta**

**hyperid** 87001

**name** log alpha

**short.name** a

**output.name** zero-probability parameter for zero-inflated poisson\_2

**output.name.intern** intern zero-probability parameter for zero-inflated poisson\_2

**initial** 0.693147180559945

**fixed** FALSE

**prior** gaussian

**param** 0.693147180559945 1

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** zeroinflated

## Zeroinflated Binomial Type 2

doc Zero-inflated Binomial, type 2

hyper

theta

hyperid 92001

name alpha

short.name alpha

output.name zero-probability parameter for zero-inflated binomial\_2

output.name.intern intern zero-probability parameter for zero-inflated binomial\_2

initial -1

fixed FALSE

prior gaussian

param -1 0.2

to.theta function(x) log(x)

from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

pdf zeroinflated

## Zeroinflated Negative Binomial Type 2

doc Zero inflated negBinomial, type 2

hyper

theta1

hyperid 99001

name log size

short.name size

output.name size for nbinoomial zero-inflated observations

output.name.inter log size for nbinoomial zero-inflated observations

initial 2.30258509299405

fixed FALSE

prior pc.mgamma

param 7

to.theta function(x) log(x)

from.theta function(x) exp(x)

theta2

hyperid 99002

name log alpha

short.name a

output.name parameter alpha for zero-inflated nbinoomial2

```

output.name.intern parameter alpha.intern for zero-inflated nbinomial2
initial 0.693147180559945
fixed FALSE
prior gaussian
param 2 1
to.theta function(x) log(x)
from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

```

## Zeroinflated Negative Binomial Type 1 Strata 2

doc Zero inflated negBinomial, type 1, strata 2

hyper

theta1

```

hyperid 97001
name log size
short.name size
output.name size for zero-inflated nbinomial_1_strata2
output.name.intern log size for zero-inflated nbinomial_1_strata2
initial 2.30258509299405
fixed FALSE
prior pc.mgamma
param 7
to.theta function(x) log(x)
from.theta function(x) exp(x)

```

theta2

```

hyperid 97002
name logit probability 1
short.name prob1
output.name zero-probability1 for zero-inflated nbinomial_1_strata2
output.name.intern intern zero-probability1 for zero-inflated nbinomial_1_strata2
initial -1
fixed FALSE
prior gaussian
param -1 0.2
to.theta function(x) log(x / (1 - x))
from.theta function(x) exp(x) / (1 + exp(x))

```

theta3

```

hyperid 97003

```

```

name logit probability 2
short.name prob2
output.name zero-probability2 for zero-inflated nbinomial_1_strata2
output.name.intern intern zero-probability2 for zero-inflated nbinomial_1_strata2
initial -1
fixed FALSE
prior gaussian
param -1 0.2
to.theta function(x) log(x / (1 - x))
from.theta function(x) exp(x) / (1 + exp(x))

theta4
  hyperid 97004
  name logit probability 3
  short.name prob3
  output.name zero-probability3 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability3 for zero-inflated nbinomial_1_strata2
  initial -1
  fixed TRUE
  prior gaussian
  param -1 0.2
  to.theta function(x) log(x / (1 - x))
  from.theta function(x) exp(x) / (1 + exp(x))

theta5
  hyperid 97005
  name logit probability 4
  short.name prob4
  output.name zero-probability4 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability4 for zero-inflated nbinomial_1_strata2
  initial -1
  fixed TRUE
  prior gaussian
  param -1 0.2
  to.theta function(x) log(x / (1 - x))
  from.theta function(x) exp(x) / (1 + exp(x))

theta6
  hyperid 97006
  name logit probability 5
  short.name prob5
  output.name zero-probability5 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability5 for zero-inflated nbinomial_1_strata2
  initial -1
  fixed TRUE
  prior gaussian
  param -1 0.2

```

```

to.theta function(x) log(x / (1 - x))
from.theta function(x) exp(x) / (1 + exp(x))
theta7
  hyperid 97007
  name logit probability 6
  short.name prob6
  output.name zero-probability6 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability6 for zero-inflated nbinomial_1_strata2
  initial -1
  fixed TRUE
  prior gaussian
  param -1 0.2
  to.theta function(x) log(x / (1 - x))
  from.theta function(x) exp(x) / (1 + exp(x))
theta8
  hyperid 97008
  name logit probability 7
  short.name prob7
  output.name zero-probability7 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability7 for zero-inflated nbinomial_1_strata2
  initial -1
  fixed TRUE
  prior gaussian
  param -1 0.2
  to.theta function(x) log(x / (1 - x))
  from.theta function(x) exp(x) / (1 + exp(x))
theta9
  hyperid 97009
  name logit probability 8
  short.name prob8
  output.name zero-probability8 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability8 for zero-inflated nbinomial_1_strata2
  initial -1
  fixed TRUE
  prior gaussian
  param -1 0.2
  to.theta function(x) log(x / (1 - x))
  from.theta function(x) exp(x) / (1 + exp(x))
theta10
  hyperid 97010
  name logit probability 9
  short.name prob9
  output.name zero-probability9 for zero-inflated nbinomial_1_strata2
  output.name.intern intern zero-probability9 for zero-inflated nbinomial_1_strata2

```

```

    initial -1
    fixed TRUE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
theta11
    hyperid 97011
    name logit probability 10
    short.name prob10
    output.name zero-probability10 for zero-inflated nbinomial_1_strata2
    output.name.intern intern zero-probability10 for zero-inflated nbinomial_1_strata2
    initial -1
    fixed TRUE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))

```

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

### Zeroinflated Negative Binomial Type 1 Strata 3

doc Zero inflated negBinomial, type 1, strata 3

hyper

```

theta1
    hyperid 98001
    name logit probability
    short.name prob
    output.name zero-probability for zero-inflated nbinomial_1_strata3
    output.name.intern intern zero-probability for zero-inflated nbinomial_1_strata3
    initial -1
    fixed FALSE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
theta2
    hyperid 98002
    name log size 1

```



```

short.name size1
output.name size1 for zero-inflated nbinomial_1_strata3
output.name.intern log_size1 for zero-inflated nbinomial_1_strata3
initial 2.30258509299405
fixed FALSE
prior pc.mgamma
param 7
to.theta function(x) log(x)
from.theta function(x) exp(x)
theta3
  hyperid 98003
  name log size 2
  short.name size2
  output.name size2 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size2 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed FALSE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
theta4
  hyperid 98004
  name log size 3
  short.name size3
  output.name size3 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size3 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
theta5
  hyperid 98005
  name log size 4
  short.name size4
  output.name size4 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size4 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)

```

```

    from.theta function(x) exp(x)
theta6
  hyperid 98006
  name log size 5
  short.name size5
  output.name size5 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size5 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
theta7
  hyperid 98007
  name log size 6
  short.name size6
  output.name size6 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size6 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
theta8
  hyperid 98008
  name log size 7
  short.name size7
  output.name size7 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size7 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
theta9
  hyperid 98009
  name log size 8
  short.name size8
  output.name size8 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size8 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405

```

```

fixed TRUE
prior pc.mgamma
param 7
to.theta function(x) log(x)
from.theta function(x) exp(x)
theta10
  hyperid 98010
  name log size 9
  short.name size9
  output.name size9 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size9 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
theta11
  hyperid 98011
  name log size 10
  short.name size10
  output.name size10 for zero-inflated nbinomial_1_strata3
  output.name.intern log_size10 for zero-inflated nbinomial_1_strata3
  initial 2.30258509299405
  fixed TRUE
  prior pc.mgamma
  param 7
  to.theta function(x) log(x)
  from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

```

### 0.0.1 Zero and $N$ -inflated Binomial likelihood: type 3

This is the case where

$$\begin{aligned}
 \text{Prob}(y|\dots) &= p_0 \times 1_{[y=0]} + \\
 &\quad p_N \times 1_{[y=N]} + \\
 &\quad (1 - p_0 - p_N) \times \text{binomial}(y, N, p)
 \end{aligned}$$

where:

$$p = \frac{\exp(\eta)}{1 + \exp(\eta)} \quad p_0 = \frac{p^{\alpha_0}}{1 + p^{\alpha_0} + (1 - p)^{\alpha_N}} \quad p_N = \frac{(1 - p)^{\alpha_N}}{1 + p^{\alpha_0} + (1 - p)^{\alpha_N}}$$

There are 2 hyperparameters,  $\alpha_0$  and  $\alpha_N$ , governing zero-inflation where: The zero-inflation parameters  $\alpha_0$  and  $\alpha_N$  are represented as  $\theta_0 = \log(\alpha_0)$ ;  $\theta_N = \log(\alpha_N)$  and the prior and initial value is given for  $\theta_0$  and  $\theta_N$  respectively.

Here is an example

```
nsim<-10000
x<-rnorm(nsim)
alpha0<-1.5
alphaN<-2.0
p = exp(x)/(1+exp(x))
p0 = p^alpha0 / (1 + p^alpha0 + (1-p)^alphaN)
pN = (1-p)^alphaN / (1 + p^alpha0 + (1-p)^alphaN)
P<-cbind(p0, pN, (1-p0 -pN))
N<-rpois(nsim,20)
y<-rep(0,nsim)
for(i in 1:nsim)
  y[i]<-sum(rmultinom(1,size = 1,P[i,])*c(0,N[i],rbinom(1,N[i],p[i])))
formula = y ~1 + x
r = inla(formula, family = "zeroninflatedbinomial3", Ntrials = N, verbose = TRUE,
  data = data.frame(y, x))
```

and the default settings

**doc** Zero and N inflated binomial, type 3

**hyper**

**theta1**

```
hyperid 93101
name alpha0
short.name alpha0
output.name alpha0 parameter for zero-n-inflated binomial_3
output.name.intern intern alpha0 parameter for zero-n-inflated binomial_3
initial 1
fixed FALSE
prior loggamma
param 1 1
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

**theta2**

```
hyperid 93102
name alphaN
short.name alphaN
output.name.intern intern alphaN parameter for zero-n-inflated binomial_3
output.name alphaN parameter for zero-n-inflated binomial_3
initial 1
fixed FALSE
prior loggamma
param 1 1
```

```
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

```
survival FALSE
```

```
discrete FALSE
```

```
link default logit loga cauchit probit cloglog ccloglog loglog robit sn
```

```
pdf zeroinflated
```