

## Special/uncommon versions of the Poisson

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!} \exp(-\lambda)$$

for responses  $y = 0, 1, 2, \dots$ , where

$\lambda$ : the expected value.

Various special versions can be defined from here.

### Special1

Here we consider a 1-inflated modification where there its known there are no zeros

$$\text{Prob}(y) = p \times 1_{[y=1]} + \frac{1-p}{1-\exp(-\lambda)} \times \frac{\lambda^y}{y!} \exp(-\lambda), \quad y = 1, 2, \dots$$

#### Link-function

$\lambda$  is linked to the linear predictor by

$$\lambda = E \exp(\eta)$$

where  $E > 0$  is a known constant (or  $\log(E)$  is the offset of  $\eta$ ).

#### Hyperparameters

**Special1** have hyperparameter  $p$  which is represented as

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior is given on  $\theta$

**doc** The Poisson.special1 likelihood

**hyper**

**theta**

**hyperid** 56100

**name** logit probability

**short.name** prob

**initial** -1

**fixed** FALSE

**prior** gaussian

**param** -1 0.2

**to.theta** function(x) log(x / (1 - x))

**from.theta** function(x) exp(x) / (1 + exp(x))

**survival** FALSE

**discrete** TRUE

**link** default log

**pdf** poisson-special

## Specification

- family = poisson.special1
- Required arguments: (integer-valued)  $y$  and  $E$

## Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n <- 300
a <- 1
b <- 1
p <- 0.2
x <- rnorm(n, sd = 0.2)
mu <- exp(a+b*x)
y.max <- ceiling(max(mu + 10*sqrt(mu)))
y <- numeric(n)

for(i in 1:n) {
  yy <- 1:y.max
  dy <- dpois(yy, lambda = mu[i])
  dy <- dy/sum(dy)
  dy <- dy * (1-p)
  dy[1] <- dy[1] + p
  y[i] <- sample(x = yy, size = 1, prob = dy)
}

r <- inla(y ~ 1 + x,
          data = data.frame(y, x),
          family = "poisson.special1")
summary(r)
```

## Notes