

## Correlated random effects: iidkd

This model is available for dimensions  $k = 2$ , to 20. We describe in detail the case for  $k = 3$  as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a different and more efficient parameterisation.

### Parametrization

The  $(k = 3)$ -dimensional Normal-Wishard model is used if one want to define three vectors of “random effects”,  $u$  and  $v$  and  $w$ , say, for which  $(u_i, v_i, w_i)$  are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

where the covariance matrix  $\mathbf{W}^{-1}$  is parameterised as  $\mathbf{W} = \mathbf{L}\mathbf{L}^T$ , where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \quad (1)$$

and  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$  can take any value. The number of hyperparameters are  $k(k+1)/2$ , which is 3, 6, 10, 15, 21, 28, 36, 45, 55, ..., for  $k = 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots$

For these models the precision matrix  $\mathbf{W}$  is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(k+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > k+1$$

and

$$c = 2^{(rk)/2} |\mathbf{R}|^{-r/2} \pi^{(k(k-1))/4} \prod_{j=1}^k \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (k+1)).$$

### Hyperparameters

The hyperparameters are  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ .

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters  $\{\theta_i\}$ , and the conversion into interpretable quantities can be done using simulation as described below.

The prior for  $\theta$  is **fixed** to be `wishartkd`, and number of prior parameters required are  $1 + k(k+1)/2$ . By default the prior-parameters are

$$(r = 100, \underbrace{1, \dots, 1}_{k \text{ times}}, 0, \dots, 0)$$

The model `iidkd` is specified as

where  $\text{order} = k = 3$ , and the `iidkd` model is represented internally as one vector of length  $n$ ,

where  $n = 3m$ , and  $n$  is the (required) argument in  $\mathbf{f}()$ .

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

(**Note:** The value “1048576” is just a code for “replace this by the default value”. As the default value depends on **order**, this was the easy way out for the moment.)

hyper

hyperid 29101

short.name theta1

fixed FALSE

[illegible]

```

    to.theta function(x) x
    from.theta function(x) x
theta2
  hyperid 29102
  name theta2
  short.name theta2
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta3
  hyperid 29103
  name theta3
  short.name theta3
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta4
  hyperid 29104
  name theta4
  short.name theta4
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta5
  hyperid 29105
  name theta5
  short.name theta5
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta6
  hyperid 29106

```

```

    name theta6
    short.name theta6
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 29107
    name theta7
    short.name theta7
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 29108
    name theta8
    short.name theta8
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 29109
    name theta9
    short.name theta9
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 29110
    name theta10
    short.name theta10
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 29111
    name theta11
    short.name theta11
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta12
    hyperid 29112
    name theta12
    short.name theta12
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta13
    hyperid 29113
    name theta13
    short.name theta13
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta14
    hyperid 29114
    name theta14
    short.name theta14
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta15
  hyperid 29115
  name theta15
  short.name theta15
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta16
  hyperid 29116
  name theta16
  short.name theta16
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta17
  hyperid 29117
  name theta17
  short.name theta17
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta18
  hyperid 29118
  name theta18
  short.name theta18
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta19
  hyperid 29119
  name theta19
  short.name theta19

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta20
    hyperid 29120
    name theta20
    short.name theta20
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta21
    hyperid 29121
    name theta21
    short.name theta21
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta22
    hyperid 29122
    name theta22
    short.name theta22
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta23
    hyperid 29123
    name theta23
    short.name theta23
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta24
  hyperid 29124
  name theta24
  short.name theta24
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta25
  hyperid 29125
  name theta25
  short.name theta25
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta26
  hyperid 29126
  name theta26
  short.name theta26
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta27
  hyperid 29127
  name theta27
  short.name theta27
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta28
  hyperid 29128

```



```

    name theta28
    short.name theta28
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta29
    hyperid 29129
    name theta29
    short.name theta29
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta30
    hyperid 29130
    name theta30
    short.name theta30
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 29131
    name theta31
    short.name theta31
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 29132
    name theta32
    short.name theta32
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 29133
    name theta33
    short.name theta33
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta34
    hyperid 29134
    name theta34
    short.name theta34
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta35
    hyperid 29135
    name theta35
    short.name theta35
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta36
    hyperid 29136
    name theta36
    short.name theta36
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

#### theta37

hyperid 29137  
name theta37  
short.name theta37  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta38

hyperid 29138  
name theta38  
short.name theta38  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta39

hyperid 29139  
name theta39  
short.name theta39  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta40

hyperid 29140  
name theta40  
short.name theta40  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta41

hyperid 29141  
name theta41  
short.name theta41

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 29142
    name theta42
    short.name theta42
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta43
    hyperid 29143
    name theta43
    short.name theta43
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 29144
    name theta44
    short.name theta44
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 29145
    name theta45
    short.name theta45
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta46
  hyperid 29146
  name theta46
  short.name theta46
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta47
  hyperid 29147
  name theta47
  short.name theta47
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta48
  hyperid 29148
  name theta48
  short.name theta48
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta49
  hyperid 29149
  name theta49
  short.name theta49
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta50
  hyperid 29150

```

```

    name theta50
    short.name theta50
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 29151
    name theta51
    short.name theta51
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta52
    hyperid 29152
    name theta52
    short.name theta52
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta53
    hyperid 29153
    name theta53
    short.name theta53
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta54
    hyperid 29154
    name theta54
    short.name theta54
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta55
    hyperid 29155
    name theta55
    short.name theta55
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta56
    hyperid 29156
    name theta56
    short.name theta56
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta57
    hyperid 29157
    name theta57
    short.name theta57
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta58
    hyperid 29158
    name theta58
    short.name theta58
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

#### theta59

hyperid 29159  
name theta59  
short.name theta59  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta60

hyperid 29160  
name theta60  
short.name theta60  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta61

hyperid 29161  
name theta61  
short.name theta61  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta62

hyperid 29162  
name theta62  
short.name theta62  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta63

hyperid 29163  
name theta63  
short.name theta63



```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta64
    hyperid 29164
    name theta64
    short.name theta64
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta65
    hyperid 29165
    name theta65
    short.name theta65
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta66
    hyperid 29166
    name theta66
    short.name theta66
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta67
    hyperid 29167
    name theta67
    short.name theta67
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta68
  hyperid 29168
  name theta68
  short.name theta68
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta69
  hyperid 29169
  name theta69
  short.name theta69
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta70
  hyperid 29170
  name theta70
  short.name theta70
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta71
  hyperid 29171
  name theta71
  short.name theta71
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta72
  hyperid 29172

```

```

    name theta72
    short.name theta72
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta73
    hyperid 29173
    name theta73
    short.name theta73
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta74
    hyperid 29174
    name theta74
    short.name theta74
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta75
    hyperid 29175
    name theta75
    short.name theta75
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta76
    hyperid 29176
    name theta76
    short.name theta76
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta77
    hyperid 29177
    name theta77
    short.name theta77
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta78
    hyperid 29178
    name theta78
    short.name theta78
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta79
    hyperid 29179
    name theta79
    short.name theta79
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta80
    hyperid 29180
    name theta80
    short.name theta80
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta81
  hyperid 29181
  name theta81
  short.name theta81
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta82
  hyperid 29182
  name theta82
  short.name theta82
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta83
  hyperid 29183
  name theta83
  short.name theta83
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta84
  hyperid 29184
  name theta84
  short.name theta84
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta85
  hyperid 29185
  name theta85
  short.name theta85

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta86
    hyperid 29186
    name theta86
    short.name theta86
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta87
    hyperid 29187
    name theta87
    short.name theta87
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta88
    hyperid 29188
    name theta88
    short.name theta88
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta89
    hyperid 29189
    name theta89
    short.name theta89
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta90
  hyperid 29190
  name theta90
  short.name theta90
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta91
  hyperid 29191
  name theta91
  short.name theta91
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta92
  hyperid 29192
  name theta92
  short.name theta92
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta93
  hyperid 29193
  name theta93
  short.name theta93
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta94
  hyperid 29194

```

```

    name theta94
    short.name theta94
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta95
    hyperid 29195
    name theta95
    short.name theta95
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta96
    hyperid 29196
    name theta96
    short.name theta96
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta97
    hyperid 29197
    name theta97
    short.name theta97
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta98
    hyperid 29198
    name theta98
    short.name theta98
    initial 1048576
    fixed FALSE

```



```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta99
    hyperid 29199
    name theta99
    short.name theta99
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta100
    hyperid 29200
    name theta100
    short.name theta100
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta101
    hyperid 29201
    name theta101
    short.name theta101
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta102
    hyperid 29202
    name theta102
    short.name theta102
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

#### theta103

hyperid 29203  
name theta103  
short.name theta103  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta104

hyperid 29204  
name theta104  
short.name theta104  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta105

hyperid 29205  
name theta105  
short.name theta105  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta106

hyperid 29206  
name theta106  
short.name theta106  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta107

hyperid 29207  
name theta107  
short.name theta107

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta108
    hyperid 29208
    name theta108
    short.name theta108
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta109
    hyperid 29209
    name theta109
    short.name theta109
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta110
    hyperid 29210
    name theta110
    short.name theta110
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta111
    hyperid 29211
    name theta111
    short.name theta111
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta112
  hyperid 29212
  name theta112
  short.name theta112
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta113
  hyperid 29213
  name theta113
  short.name theta113
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta114
  hyperid 29214
  name theta114
  short.name theta114
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta115
  hyperid 29215
  name theta115
  short.name theta115
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta116
  hyperid 29216

```

```

    name theta116
    short.name theta116
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta117
    hyperid 29217
    name theta117
    short.name theta117
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta118
    hyperid 29218
    name theta118
    short.name theta118
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta119
    hyperid 29219
    name theta119
    short.name theta119
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta120
    hyperid 29220
    name theta120
    short.name theta120
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta121
    hyperid 29221
    name theta121
    short.name theta121
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta122
    hyperid 29222
    name theta122
    short.name theta122
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta123
    hyperid 29223
    name theta123
    short.name theta123
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta124
    hyperid 29224
    name theta124
    short.name theta124
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta125
  hyperid 29225
  name theta125
  short.name theta125
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta126
  hyperid 29226
  name theta126
  short.name theta126
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta127
  hyperid 29227
  name theta127
  short.name theta127
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta128
  hyperid 29228
  name theta128
  short.name theta128
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta129
  hyperid 29229
  name theta129
  short.name theta129

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta130
    hyperid 29230
    name theta130
    short.name theta130
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta131
    hyperid 29231
    name theta131
    short.name theta131
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta132
    hyperid 29232
    name theta132
    short.name theta132
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta133
    hyperid 29233
    name theta133
    short.name theta133
    initial 1048576
    fixed FALSE
    prior none
    param

```



```

    to.theta function(x) x
    from.theta function(x) x
theta134
  hyperid 29234
  name theta134
  short.name theta134
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta135
  hyperid 29235
  name theta135
  short.name theta135
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta136
  hyperid 29236
  name theta136
  short.name theta136
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta137
  hyperid 29237
  name theta137
  short.name theta137
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta138
  hyperid 29238

```

```

    name theta138
    short.name theta138
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta139
    hyperid 29239
    name theta139
    short.name theta139
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta140
    hyperid 29240
    name theta140
    short.name theta140
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta141
    hyperid 29241
    name theta141
    short.name theta141
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta142
    hyperid 29242
    name theta142
    short.name theta142
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta143
    hyperid 29243
    name theta143
    short.name theta143
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta144
    hyperid 29244
    name theta144
    short.name theta144
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta145
    hyperid 29245
    name theta145
    short.name theta145
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta146
    hyperid 29246
    name theta146
    short.name theta146
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta147
  hyperid 29247
  name theta147
  short.name theta147
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta148
  hyperid 29248
  name theta148
  short.name theta148
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta149
  hyperid 29249
  name theta149
  short.name theta149
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta150
  hyperid 29250
  name theta150
  short.name theta150
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta151
  hyperid 29251
  name theta151
  short.name theta151

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta152
    hyperid 29252
    name theta152
    short.name theta152
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta153
    hyperid 29253
    name theta153
    short.name theta153
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta154
    hyperid 29254
    name theta154
    short.name theta154
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta155
    hyperid 29255
    name theta155
    short.name theta155
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta156
  hyperid 29256
  name theta156
  short.name theta156
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta157
  hyperid 29257
  name theta157
  short.name theta157
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta158
  hyperid 29258
  name theta158
  short.name theta158
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta159
  hyperid 29259
  name theta159
  short.name theta159
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta160
  hyperid 29260

```

```

    name theta160
    short.name theta160
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta161
    hyperid 29261
    name theta161
    short.name theta161
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta162
    hyperid 29262
    name theta162
    short.name theta162
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta163
    hyperid 29263
    name theta163
    short.name theta163
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta164
    hyperid 29264
    name theta164
    short.name theta164
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta165
    hyperid 29265
    name theta165
    short.name theta165
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta166
    hyperid 29266
    name theta166
    short.name theta166
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta167
    hyperid 29267
    name theta167
    short.name theta167
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta168
    hyperid 29268
    name theta168
    short.name theta168
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```



**theta169**

hyperid 29269  
name theta169  
short.name theta169  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta170**

hyperid 29270  
name theta170  
short.name theta170  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta171**

hyperid 29271  
name theta171  
short.name theta171  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta172**

hyperid 29272  
name theta172  
short.name theta172  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta173**

hyperid 29273  
name theta173  
short.name theta173

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta174
    hyperid 29274
    name theta174
    short.name theta174
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta175
    hyperid 29275
    name theta175
    short.name theta175
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta176
    hyperid 29276
    name theta176
    short.name theta176
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta177
    hyperid 29277
    name theta177
    short.name theta177
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta178
  hyperid 29278
  name theta178
  short.name theta178
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta179
  hyperid 29279
  name theta179
  short.name theta179
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta180
  hyperid 29280
  name theta180
  short.name theta180
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta181
  hyperid 29281
  name theta181
  short.name theta181
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta182
  hyperid 29282

```

```

    name theta182
    short.name theta182
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta183
    hyperid 29283
    name theta183
    short.name theta183
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta184
    hyperid 29284
    name theta184
    short.name theta184
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta185
    hyperid 29285
    name theta185
    short.name theta185
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta186
    hyperid 29286
    name theta186
    short.name theta186
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta187
    hyperid 29287
    name theta187
    short.name theta187
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta188
    hyperid 29288
    name theta188
    short.name theta188
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta189
    hyperid 29289
    name theta189
    short.name theta189
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta190
    hyperid 29290
    name theta190
    short.name theta190
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta191
  hyperid 29291
  name theta191
  short.name theta191
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta192
  hyperid 29292
  name theta192
  short.name theta192
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta193
  hyperid 29293
  name theta193
  short.name theta193
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta194
  hyperid 29294
  name theta194
  short.name theta194
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta195
  hyperid 29295
  name theta195
  short.name theta195

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta196
    hyperid 29296
    name theta196
    short.name theta196
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta197
    hyperid 29297
    name theta197
    short.name theta197
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta198
    hyperid 29298
    name theta198
    short.name theta198
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta199
    hyperid 29299
    name theta199
    short.name theta199
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta200
  hyperid 29300
  name theta200
  short.name theta200
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta201
  hyperid 29301
  name theta201
  short.name theta201
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta202
  hyperid 29302
  name theta202
  short.name theta202
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta203
  hyperid 29303
  name theta203
  short.name theta203
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta204
  hyperid 29304

```



```

    name theta204
    short.name theta204
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta205
    hyperid 29305
    name theta205
    short.name theta205
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta206
    hyperid 29306
    name theta206
    short.name theta206
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta207
    hyperid 29307
    name theta207
    short.name theta207
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta208
    hyperid 29308
    name theta208
    short.name theta208
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
  theta209
    hyperid 29309
    name theta209
    short.name theta209
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
  theta210
    hyperid 29310
    name theta210
    short.name theta210
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

  constr FALSE

  nrow.ncol FALSE

  augmented TRUE

  aug.factor 1

  aug.constr 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

  n.div.by -1

  n.required TRUE

  set.default.values TRUE

  pdf iidkd

```

## Example

Just simulate some data and estimate the parameters back. This is for `order=4`.

```

library(mvtnorm)

n <- 300
m <- 4

```

```

N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
  for (j in 1:m) {
    if (i != j) {
      Sigma[i, j] <- rho^abs(i-j) * sqrt(Sigma[i, i] * Sigma[j, j])
    }
  }
}

y <- c()
yy <- rmvnorm(n, sigma = Sigma)
for(i in 1:m) {
  y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
  ## set parameters using 'theta1'.
  ## these are the default parameters.
  hyper = list(theta1 = list(
    param = c(100, rep(1, m), rep(0, m*(m-1)/2))))),
  data = data.frame(i = 1:N, y),
  ## fix precision as we have exact observations
  control.family = list(hyper = list(
    prec = list(initial = 15, fixed = TRUE))),
  verbose = FALSE)

## this is how the internal parameters are defined
L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))

## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
  mle = c(diag(LL), LL[lower.tri(LL)]),
  inla = r$mode$theta))

## this gives a list of sampled matrices (stdev's and correlations)
xx <- inla.iidkd.sample(10^4, r, "i")
## compute the mean
qq <- matrix(rowMeans(matrix(unlist(xx), nrow = m^2)), m, m)

iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(diag(qq), qq[lower.tri(qq)]),
  true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))

```

## Notes

Note that for higher values of  $k$  the only integration scheme that is supported is `control.inla=list(int.strategy="eb")`.