

## NMix

### Parametrisation

The N-Mixture distribution is a Poisson mixture of the Binomials, as

$$\text{Prob}(y) = \sum_{n=y}^{\infty} \binom{n}{y} p^n (1-p)^{n-y} \times \frac{\lambda^n}{n!} \exp(-\lambda)$$

for responses  $y = 0, 1, 2, \dots, n$ , where  $n$  is Poisson number of trials, and  $p$  is probability of success. Replicated responses  $y_1, y_2, \dots, y_r$ , are iid from the Binomial with the same  $p$ , conditioned on the same  $n$  from the Poisson,

$$\text{Prob}(y_1, \dots, y_r) = \sum_{n=\max\{y_1, \dots, y_r\}}^{\infty} \left\{ \prod_{i=1}^r \binom{n}{y_i} p^n (1-p)^{n-y_i} \right\} \times \frac{\lambda^n}{n!} \exp(-\lambda)$$

### Link-function

The probability  $p$  is linked to the linear predictor by

$$p(\eta) = \frac{\exp(\eta)}{1 + \exp(\eta)}$$

for the default logit link, while  $\lambda$  depends on fixed covariates

$$\log(\lambda) = \sum_{j=1}^m \beta_j x_j$$

with one vector of covariates for each observation.  $m$  can be maximum 15 and minimum 1.

### Hyperparameters

The parameters  $\theta_1 = \beta_1, \theta_2 = \beta_2, \dots, \theta_m = \beta_m$ .

### Hyperparameter specification and default values

**doc** Binomial-Poisson mixture

**hyper**

**theta1**

**hyperid** 101101

**name** beta1

**short.name** beta1

**output.name** beta[1] for NMix observations

**output.name.intern** beta[1] for NMix observations

**initial** 2.30258509299405

**fixed** FALSE

**prior** normal

**param** 0 0.5

**to.theta** function(x) x

**from.theta** function(x) x

#### theta2

```
hyperid 101102
name beta2
short.name beta2
output.name beta[2] for NMix observations
output.name.intern beta[2] for NMix observations
initial 0
fixed FALSE
prior normal
param 0 1
to.theta function(x) x
from.theta function(x) x
```

#### theta3

```
hyperid 101103
name beta3
short.name beta3
output.name beta[3] for NMix observations
output.name.intern beta[3] for NMix observations
initial 0
fixed FALSE
prior normal
param 0 1
to.theta function(x) x
from.theta function(x) x
```

#### theta4

```
hyperid 101104
name beta4
short.name beta4
output.name beta[4] for NMix observations
output.name.intern beta[4] for NMix observations
initial 0
fixed FALSE
prior normal
param 0 1
to.theta function(x) x
from.theta function(x) x
```

#### theta5

```
hyperid 101105
name beta5
short.name beta5
output.name beta[5] for NMix observations
output.name.intern beta[5] for NMix observations
initial 0
fixed FALSE
```

```

    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 101106
    name beta6
    short.name beta6
    output.name beta[6] for NMix observations
    output.name.intern beta[6] for NMix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 101107
    name beta7
    short.name beta7
    output.name beta[7] for NMix observations
    output.name.intern beta[7] for NMix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 101108
    name beta8
    short.name beta8
    output.name beta[8] for NMix observations
    output.name.intern beta[8] for NMix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 101109
    name beta9
    short.name beta9

```

```

output.name beta[9] for NMix observations
output.name.intern beta[9] for NMix observations
initial 0
fixed FALSE
prior normal
param 0 1
to.theta function(x) x
from.theta function(x) x
theta10
  hyperid 101110
  name beta10
  short.name beta10
  output.name beta[10] for NMix observations
  output.name.intern beta[10] for NMix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 1
  to.theta function(x) x
  from.theta function(x) x
theta11
  hyperid 101111
  name beta11
  short.name beta11
  output.name beta[11] for NMix observations
  output.name.intern beta[11] for NMix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 1
  to.theta function(x) x
  from.theta function(x) x
theta12
  hyperid 101112
  name beta12
  short.name beta12
  output.name beta[12] for NMix observations
  output.name.intern beta[12] for NMix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 1
  to.theta function(x) x
  from.theta function(x) x

```

theta13

hyperid 101113  
name beta13  
short.name beta13  
output.name beta[13] for NMix observations  
output.name.intern beta[13] for NMix observations  
initial 0  
fixed FALSE  
prior normal  
param 0 1  
to.theta function(x) x  
from.theta function(x) x

theta14

hyperid 101114  
name beta14  
short.name beta14  
output.name beta[14] for NMix observations  
output.name.intern beta[14] for NMix observations  
initial 0  
fixed FALSE  
prior normal  
param 0 1  
to.theta function(x) x  
from.theta function(x) x

theta15

hyperid 101115  
name beta15  
short.name beta15  
output.name beta[15] for NMix observations  
output.name.intern beta[15] for NMix observations  
initial 0  
fixed FALSE  
prior normal  
param 0 1  
to.theta function(x) x  
from.theta function(x) x

survival FALSE

discrete TRUE

link default logit loga probit

pdf nmix

## Specification

- `family="nmix"`
- Required arguments: the response  $Y$  and covariates  $X$  as `inla.mdata(Y, X [, additional.covariates])`

The response is a matrix where each row are replicates, where responses that are NA's are ignored. The covariates is one or many vectors, matrices or data.frames. Each row of the covariates  $(x_{i1}, x_{i2}, \dots, x_{im})$  defines the covariates used for the  $i$ 'th response(s) (the  $i$ 'th row of  $Y$ ). Note that  $\beta_{m+1}, \dots, \beta_{15}$  are fixed to zero.

## Example

In the following example we estimate the parameters in a simulated example with replications.

```
nrep.max = 5
n = 50
y = matrix(NA, n, nrep.max)
x = c()
xx = c()
intercept = 1

for(i in 1:n) {
  local.x = runif(1) - 0.5
  lambda = exp(2 + local.x)
  N = rpois(1, lambda)
  local.xx = runif(1) - 0.5
  eta = intercept + local.xx
  p = exp(eta)/(exp(eta) + 1)
  ## sample the number of replications
  nr = sample(1:nrep.max, 1)
  ## and sample these. note that 'y' is initialized with NA's,
  ## so if nr < nrep.max, then
  ## y[i,(nr+1):nrep.max] would be NA.
  y[i, 1:nr]= rbinom(nr, size = N, prob = p)
  x = c(x, local.x)
  xx = c(xx, local.xx)
}

Y = inla.mdata(y, 1, x)
r = inla(Y ~ 1 + xx,
  data = list(Y=Y, xx=xx),
  family = "nmix",
  control.fixed = list(prec.intercept=1, prec=1))
```

## Notes