Generalized Poisson

The generalized Poisson distribution is given by

$$f(y|\lambda, w) = \frac{\lambda(\lambda + wy)^{y-1}}{y!} \exp(-(\lambda + wy))$$

for $y = 0, 1, 2, \ldots$ and where $\lambda > 0$ and $\max(-1, -\lambda/4) \leq w \leq 1$. The mean and variance of y are

$$\mu = \lambda(1-w)^{-1} \qquad \text{and} \qquad \sigma^2 = \lambda(1-w)^{-3} = \mu(1-w)^{-2}.$$

Since the dispersion parameter w influence the mean as well as the variance, we will use the following parameterisation (Consul and Jain (1973), Zamani and Ismail(2012))

$$w = \frac{\varphi\mu^{p-1}}{1 + \varphi\mu^{p-1}},$$

which gives the following density

$$f(y|\mu, \varphi, p) = \frac{\mu(\mu + \varphi\mu^{p-1}y)^{y-1}}{(1 + \varphi\mu^{p-1})^y y!} \exp\left(-\frac{\mu + \varphi\mu^{p-1}y}{1 + \varphi\mu^{p-1}}\right)$$

for $y = 0, 1, 2, \ldots$. We assume $\varphi \geq 0$.

Link-function

The mean and variance of y are given as

$$\mathrm{E}(y|.) = \mu \qquad \text{and} \qquad \mathrm{Var}(y|.) = \mu\left(1 + \varphi\mu^{p-1}\right)^2$$

and the mean is linked to the linear predictor by

$$\mu = E\exp(\eta)$$

Hyperparameters

The overdispersion parameter $\varphi \geq 0$ is represented as

$$\varphi = \exp(\theta_1)$$

The "shape" parameter p is represented as

$$p = \theta_2$$

Note that $\theta_2 = 1$ and `fixed = TRUE`, default. The prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

- `family="gpoisson"`

Hyperparameter spesification and default values

doc `The generalized Poisson likelihood`

hyper

theta1

- **hyperid** `56001`
- **name** `overdispersion`
- **short.name** `phi`
- **output.name** `Overdispersion for gpoisson`
- **output.name.intern** `Log overdispersion for gpoisson`
- **initial** `0`
- **fixed** `FALSE`
- **prior** `loggamma`
- **param** `1 1`
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

theta2

- **hyperid** `56002`
- **name** `p`
- **short.name** `p`
- **output.name** `Parameter p for gpoisson`
- **output.name.intern** `Parameter p_intern for gpoisson`
- **initial** `1`
- **fixed** `TRUE`
- **prior** `normal`
- **param** `1 100`
- **to.theta** `function(x) x`
- **from.theta** `function(x) x`

survival `FALSE`

discrete `TRUE`

link `default log logoffset`

pdf `gpoisson`

Example

In the following example we estimate the parameters in a simulated example with generalized Poisson responses.

```
dgpoisson = function(y, mu, phi, p)
{
    a = mu + phi * mu^(p-1.0) * y;
    b = 1. + phi * mu^(p-1.0);
    d = exp(log(mu) + (y-1.0)*log(a) -
            y*log(b) - lfactorial(y) - a/b)
    return (d)
```

```
}
rgpoisson = function(n, mu, phi, p)
{
    stopifnot(length(mu) == 1)
    s = sqrt(mu*(1+phi*mu^(p-1))^2)
    f = 20
    low = as.integer(max(0, mu - f*s))
    high = as.integer(mu + f*s)
    prob = dgpoisson(low:high, mu, phi, p)
    y = sample(low:high, n, replace=TRUE,
            prob = prob)

    return (y)
}

n = 1000
phi = 1
p = 1
mu = exp(1 + 5*(1:n)/n)

y = numeric(n)
for(i in 1:n) {
    y[i] = rgpoisson(1, mu[i], phi, p)
}

idx = (1:n)/n
r = inla(y ~ 1 + idx,  data = data.frame(y, idx),
        family = "gpoisson")
```

Notes

The parameter p is default fixed to be 1. Allowing it to be estimated jointly with the overdispersion parameter, please note the following.

- The parameter p and the overdispersion parameter are strongly correlated when estimated jointly.
- You may want to chose an informative prior for p, as the shape of the likelihood might not be want you expect for "extreme" p.
- You may experience problems in the numerical optimization (fail to converge); a more informative prior (if available) for p will help with this issue.

References

- Consul, P. C. and Jain, G.C (1973) A generalization of Poisson distribution. Technometrics 15, 791-799.
- Zamani, H. and Ismail, N. (2011). Functional form for the generalized Poisson regression model. Communication in Statistics – Theory and Methods (IN PRESS).