

# Binomial-mixture likelihood

## Parametrisation

### Details

This likelihood is a mixture of three binomials, where two of them are simple predictors and one is general. The probability for success is

$$p = w_1 p_1 + w_2 p_2 + w_3 p_3$$

where  $w_1 + w_2 + w_3 = 1$  are the positive weights,

$$\text{logit}(p_1) = \sum_{i=1}^m \beta_i z_i + \beta_{2m+1} z_{2m+1}$$

$$\text{logit}(p_2) = \sum_{i=m+1}^{2m} \beta_i z_i + \beta_{2m+1} z_{2m+2}$$

and

$$\text{logit}(p_3) = \eta + \beta_{2m+1} z_{2m+3}$$

for fixed covariates  $\{z_i\}$ . The linear predictor  $\eta$  is defined in the formula.

**Note:**  $\beta_{2m+1}$  is the *same* variable in the three expressions. The allowed range for  $m$  is  $0 \leq m \leq 25$ .

## Link-function

The link-function is given as usual, and they are all equal.

## Hyperparameters

The  $2m + 1$  regression coefficients  $\{\beta_i\}$  are treated as hyperparameters.

## Specification

- `family="binomialmix"`
- Required arguments: A  $n \times 2$  matrix  $Y$  with the observations and the number of trials  $s$ ,  $Y = (y, s)$ , a  $n \times (2m + 3)$  matrix  $Z$  with the covariates  $Z = c(z_1, \dots, z_{2m+3})$ , and a  $n \times 2$  matrix  $W$  with weights  $W = (w_1, w_2)$ . The `inla.mdata` is used as

`inla.mdata(Y, Z, W) ~ ...`

## Hyperparameter specification and default values

doc Binomial mixture

hyper

theta1

hyperid 56551  
name beta1  
short.name beta1  
output.name beta1 for binomialmix observations  
output.name.intern beta1 for binomialmix observations  
initial 0  
fixed FALSE  
prior normal  
param 0 100  
to.theta function(x) x  
from.theta function(x) x

theta2

hyperid 56552  
name beta2  
short.name beta2  
output.name beta2 for binomialmix observations  
output.name.intern beta2 for binomialmix observations  
initial 0  
fixed FALSE  
prior normal  
param 0 100  
to.theta function(x) x  
from.theta function(x) x

theta3

hyperid 56553  
name beta3  
short.name beta3  
output.name beta3 for binomialmix observations  
output.name.intern beta3 for binomialmix observations  
initial 0  
fixed FALSE  
prior normal  
param 0 100  
to.theta function(x) x  
from.theta function(x) x

theta4

hyperid 56554  
name beta4  
short.name beta4  
output.name beta4 for binomialmix observations  
output.name.intern beta4 for binomialmix observations  
initial 0  
fixed FALSE  
prior normal

```

    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 56555
    name beta5
    short.name beta5
    output.name beta5 for binomialmix observations
    output.name.intern beta5 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56556
    name beta6
    short.name beta6
    output.name beta6 for binomialmix observations
    output.name.intern beta6 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56557
    name beta7
    short.name beta7
    output.name beta7 for binomialmix observations
    output.name.intern beta7 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56558
    name beta8
    short.name beta8
    output.name beta8 for binomialmix observations
    output.name.intern beta8 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100

```

```

    to.theta function(x) x
    from.theta function(x) x
theta9
  hyperid 56559
  name beta9
  short.name beta9
  output.name beta9 for binomialmix observations
  output.name.intern beta9 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 56560
  name beta10
  short.name beta10
  output.name beta10 for binomialmix observations
  output.name.intern beta10 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta11
  hyperid 56561
  name beta11
  short.name beta11
  output.name beta11 for binomialmix observations
  output.name.intern beta11 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta12
  hyperid 56562
  name beta12
  short.name beta12
  output.name beta12 for binomialmix observations
  output.name.intern beta12 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x

```

```

    from.theta function(x) x
theta13
    hyperid 56563
    name beta13
    short.name beta13
    output.name beta13 for binomialmix observations
    output.name.intern beta13 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta14
    hyperid 56564
    name beta14
    short.name beta14
    output.name beta14 for binomialmix observations
    output.name.intern beta14 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta15
    hyperid 56565
    name beta15
    short.name beta15
    output.name beta15 for binomialmix observations
    output.name.intern beta15 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta16
    hyperid 56566
    name beta16
    short.name beta16
    output.name beta16 for binomialmix observations
    output.name.intern beta16 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

```

theta17

```
hyperid 56567
name beta17
short.name beta17
output.name beta17 for binomialmix observations
output.name.intern beta17 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
```

theta18

```
hyperid 56568
name beta18
short.name beta18
output.name beta18 for binomialmix observations
output.name.intern beta18 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
```

theta19

```
hyperid 56569
name beta19
short.name beta19
output.name beta19 for binomialmix observations
output.name.intern beta19 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
```

theta20

```
hyperid 56570
name beta20
short.name beta20
output.name beta20 for binomialmix observations
output.name.intern beta20 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
```

theta21

```

hyperid 56571
name beta21
short.name beta21
output.name beta21 for binomialmix observations
output.name.intern beta21 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta22
hyperid 56572
name beta22
short.name beta22
output.name beta22 for binomialmix observations
output.name.intern beta22 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta23
hyperid 56573
name beta23
short.name beta23
output.name beta23 for binomialmix observations
output.name.intern beta23 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta24
hyperid 56574
name beta24
short.name beta24
output.name beta24 for binomialmix observations
output.name.intern beta24 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta25
hyperid 56575

```

```

name beta25
short.name beta25
output.name beta25 for binomialmix observations
output.name.intern beta25 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta26
  hyperid 56576
  name beta26
  short.name beta26
  output.name beta26 for binomialmix observations
  output.name.intern beta26 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta27
  hyperid 56577
  name beta27
  short.name beta27
  output.name beta27 for binomialmix observations
  output.name.intern beta27 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta28
  hyperid 56578
  name beta28
  short.name beta28
  output.name beta28 for binomialmix observations
  output.name.intern beta28 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta29
  hyperid 56579
  name beta29

```



```

short.name beta29
output.name beta29 for binomialmix observations
output.name.intern beta29 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta30
  hyperid 56580
  name beta30
  short.name beta30
  output.name beta30 for binomialmix observations
  output.name.intern beta30 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta31
  hyperid 56581
  name beta31
  short.name beta31
  output.name beta31 for binomialmix observations
  output.name.intern beta31 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta32
  hyperid 56582
  name beta32
  short.name beta32
  output.name beta32 for binomialmix observations
  output.name.intern beta32 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta33
  hyperid 56583
  name beta33
  short.name beta33

```

```

output.name beta33 for binomialmix observations
output.name.intern beta33 for binomialmix observations
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta34
  hyperid 56584
  name beta34
  short.name beta34
  output.name beta34 for binomialmix observations
  output.name.intern beta34 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta35
  hyperid 56585
  name beta35
  short.name beta35
  output.name beta35 for binomialmix observations
  output.name.intern beta35 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta36
  hyperid 56586
  name beta36
  short.name beta36
  output.name beta36 for binomialmix observations
  output.name.intern beta36 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta37
  hyperid 56587
  name beta37
  short.name beta37
  output.name beta37 for binomialmix observations

```

```

    output.name.intern beta37 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta38
    hyperid 56588
    name beta38
    short.name beta38
    output.name beta38 for binomialmix observations
    output.name.intern beta38 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta39
    hyperid 56589
    name beta39
    short.name beta39
    output.name beta39 for binomialmix observations
    output.name.intern beta39 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta40
    hyperid 56590
    name beta40
    short.name beta40
    output.name beta40 for binomialmix observations
    output.name.intern beta40 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta41
    hyperid 56591
    name beta41
    short.name beta41
    output.name beta41 for binomialmix observations
    output.name.intern beta41 for binomialmix observations

```

```

    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 56592
    name beta42
    short.name beta42
    output.name beta42 for binomialmix observations
    output.name.intern beta42 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta43
    hyperid 56593
    name beta43
    short.name beta43
    output.name beta43 for binomialmix observations
    output.name.intern beta43 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 56594
    name beta44
    short.name beta44
    output.name beta44 for binomialmix observations
    output.name.intern beta44 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 56595
    name beta45
    short.name beta45
    output.name beta45 for binomialmix observations
    output.name.intern beta45 for binomialmix observations
    initial 0

```

```

fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta46
  hyperid 56596
  name beta46
  short.name beta46
  output.name beta46 for binomialmix observations
  output.name.intern beta46 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta47
  hyperid 56597
  name beta47
  short.name beta47
  output.name beta47 for binomialmix observations
  output.name.intern beta47 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta48
  hyperid 56598
  name beta48
  short.name beta48
  output.name beta48 for binomialmix observations
  output.name.intern beta48 for binomialmix observations
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta49
  hyperid 56599
  name beta49
  short.name beta49
  output.name beta49 for binomialmix observations
  output.name.intern beta49 for binomialmix observations
  initial 0
  fixed FALSE

```

```

    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta50
    hyperid 56600
    name beta50
    short.name beta50
    output.name beta50 for binomialmix observations
    output.name.intern beta50 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 56601
    name beta51
    short.name beta51
    output.name beta51 for binomialmix observations
    output.name.intern beta51 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

survival FALSE
discrete TRUE
link default logit probit
pdf binomialmix

```

## Example

Here is a simple example.

```
n <- 10^5
size <- sample(5:10, n, replace = TRUE)

m <- 5
m2 <- 2*m
beta.p1 <- rnorm(m, sd = 1/sqrt(m))
beta.p2 <- rnorm(m, sd = 1/sqrt(m))
beta.common <- rnorm(1, sd = 0.5)
beta <- c(beta.p1, beta.p2, beta.common)

Z <- matrix(NA, n, m2+3)
W <- matrix(NA, n, 2)
Y <- matrix(NA, n, 2)

x <- rnorm(n, sd = 0.5)
xx <- rnorm(n, sd = 0.5)
eta <- numeric(n)

for (i in 1:n) {
  Z[i, ] <- rnorm(m2+3)
  w <- c(rbeta(2, 1, 10), rbeta(1, 10, 1))
  w <- w/sum(w)
  W[i, ] <- w[1:2]

  p1 <- inla.link.invlogit(sum(beta.p1 * Z[i, seq_len(m)])) + beta.common * Z[i, m2+1])
  p2 <- inla.link.invlogit(sum(beta.p2 * Z[i, m + seq_len(m)])) + beta.common * Z[i, m2+2])

  eta[i] <- 1 + x[i] + xx[i] + beta.common * Z[i, m2+3]
  p3 <- inla.link.invlogit(eta[i])

  p <- w[1] * p1 + w[2] * p2 + w[3] * p3
  Y[i, ] <- c(rbinom(1, size = size[i], prob = p), size[i])
}

r <- inla(inla.mdata(Y, Z, W) ~ 1 + x + xx,
  family = "binomialmix",
  data = list(Y = Y, Z = Z, W = W, x = x, xx = xx),
  verbose = TRUE,
  control.inla = list(int.strategy = "eb"))
## r <- inla.rerun(r)

print(round(dig = 4, cbind(estimate = r$summary.fixed[, "mean"], true = 1)))
print(round(dig = 4, cbind(estimate = r$summary.hyperpar[, "mean"], true = beta)))
```