

!!!!NOT USED ANYMORE!!!

## The Wishart model for correlated effects

This model is available for dimensions  $p = 2, 3$ , we describe in detail the case for  $p = 2$  and the case for  $p = 3$ .

### Parametrization

The 2-dimensional Wishart model is used if one wants to define the model for the linear predictor  $\eta$  as:

$$\eta = a + b$$

where  $a$  and  $b$  are correlated

$$\begin{pmatrix} a \\ b \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

with covariance matrix  $\mathbf{W}^{-1}$

$$\mathbf{W}^{-1} = \begin{pmatrix} 1/\tau_a & \rho\sqrt{\tau_a\tau_b} \\ \rho\sqrt{\tau_a\tau_b} & 1/\tau_b \end{pmatrix} \quad (1)$$

and  $\tau_a$ ,  $\tau_b$  and  $\rho$  are the hyperparameters. In this case the following model is implemented for the precision matrix  $\mathbf{W}$

$$\mathbf{W} \sim \text{Wishart}_p(r, \mathbf{R}^{-1}), \quad p = 2$$

where the Wishart distribution has density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(p+1))/2} \exp\left\{-\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > p + 1$$

and

$$c = 2^{(rp)/2} |\mathbf{R}|^{-r/2} \pi^{(p(p-1))/4} \prod_{j=1}^p \Gamma((r+1-j)/2).$$

Then,

$$\mathbb{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \mathbb{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (p + 1)).$$

### Hyperparameters

The hyperparameters are

$$\theta = (\log \tau_a, \log \tau_b, \tilde{\rho})$$

where

$$\rho = 2 \frac{\exp(\tilde{\rho})}{\exp(\tilde{\rho}) + 1} - 1$$

The prior-parameters are

$$(r, R_{11}, R_{22}, R_{12})$$

where

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and  $r_{12} = R_{21}$  due to symmetry.

The `inla` function reports posterior distribution for the hyperparameters  $\tau_a, \tau_b, \rho$  in equation (1).

The prior for  $\theta$  is **fixed** to be **wishart**

## Specification

The model 2d wishart for

$$\eta = a + b$$

is specified as

```
y~f(a,model="2diidwishartp1",param=<param.vector(4 elements)>)+f(b,model="2diidwishartp2")
```

The parameters for the Wishart distribution are specified *only* for 2diidwishartp1

## Example

In this example we implement the model

$$y|\eta \sim \text{Pois}(\exp(\eta))$$

where

$$\eta = a + b + c$$

and  $b$  and  $c$  are correlated as described above.

```
n=100
#set hyperparameters
r=4
R11=1
R22=2
R12=0.1
R=matrix(c(R11,R12,R12,R22),2,2)
S=solve(R)
#these are needed to simulate from a wishart prior
# and sample from a multivariate normal
library(MCMCpack)
library(mvtnorm)
W=rwish(r, S)
cc=rmvnorm(n, mean=c(0,0), sigma=solve(W))
a=cc[,1]
b=cc[,2]
#simulate data
x1=1:n
x2=1:n
eta=0.1+a+b
y=rpois(n,exp(eta))
data=data.frame(y=y,x1=1:n,x2=1:n)
#fit the model
formula=y~f(x1,model="2diidwishartp1",param=c(4,1,2,0.1))+f(x2,model="2diidwishartp2")
result=inla(formula,family="poisson",data=data)
```

## Notes

If more than one pair of `2diidwishartp1/2` is defined, the following rule is used to determine the match between `p(art)1` and `p(art)2`.

The first occurrence of `2diidwishartp1` belongs with the first occurrence of `2diidwishartp2`.

The second occurrence of `2diidwishartp1` belongs with the second occurrence of `2diidwishartp2` and so on.

## Three dimensional case

The previous formulation is also available for 3D. In this case the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23})$$

In this case the name the prior is **fixed** to be `Wishart3d`. The parameters in the prior are

$$parameters = r \ R_{11} \ R_{22} \ R_{33} \ R_{12} \ R_{13} \ R_{23}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{23} \\ R_{13} & R_{23} & R_{33} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions  $\tau_1$ ,  $\tau_2$  and  $\tau_3$  and the correlations  $\rho_{12}$ ,  $\rho_{13}$  and  $\rho_{23}$ . The model names are as given in the following example.

```
formula2 <- Y ~ f(diid.part0,model="3diidwishartp1",
  param=c(7,1,2,3,0.1,0.2,0.3))
  f(diid.part1,model="3diidwishartp2") +
  f(diid.part2,model="3diidwishartp3") +
```

If more than one pair of `3diidwishartp1/2/3` is defined, the following rule is used to determine the match between `p(art)1`, `p(art)2` and `p(art)3`.

The first occurrence of `3diidwishartp1` belongs with the first occurrence of `3diidwishartp2` and `3diidwishartp3`. The second occurrence of `3diidwishartp1` belongs with the second occurrence of `3diidwishartp2` and the second occurrence of `3diidwishartp3`, and so on.