

## Correlated random effects: iidkd

This model is available for dimensions  $k = 2$ , to 10. We describe in detail the case for  $k = 3$  as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a different and more efficient parameterisation.

### Parametrization

The  $(k = 3)$ -dimensional Normal-Wishard model is used if one want to define three vectors of “random effects”,  $u$  and  $v$  and  $w$ , say, for which  $(u_i, v_i, w_i)$  are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

where the covariance matrix  $\mathbf{W}^{-1}$  is parameterised as  $\mathbf{W} = \mathbf{L}\mathbf{L}^T$ , where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \quad (1)$$

and  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$  can take any value. The number of hyperparameters are  $k(k+1)/2$ , which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for  $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$ .

For these models the precision matrix  $\mathbf{W}$  is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(k+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > k+1$$

and

$$c = 2^{(rk)/2} |\mathbf{R}|^{-r/2} \pi^{(k(k-1))/4} \prod_{j=1}^k \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (k+1)).$$

### Hyperparameters

The hyperparameters are  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ .

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters  $\{\theta_i\}$ , and the conversion into interpretable quantities can be done using simulation as described below.

The prior for  $\theta$  is **fixed** to be `wishartkd`, and number of prior parameters required are  $1 + k(k+1)/2$ . By default the prior-parameters are

$$(r = 100, \underbrace{1, \dots, 1}_{k \text{ times}}, 0, \dots, 0)$$

## Specification

The model `iidkd` is specified as

```
y ~ f(i, model="iidkd", order=3, n = <length>) + ...
```

where  $\text{order} = k = 3$ , and the `iidkd` model is represented internally as one vector of length  $n$ ,

$$(u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_m, w_1, w_2, \dots, w_m)$$

where  $n = 3m$ , and  $n$  is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as 3 sum-to-zero constraints

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

## Hyperparameter spesification and default values

(**Note:** The value “1048576” is just a code for “replace this by the default value”. As the default value depends on `order`, the was the easy way out for the moment.)

**doc** Gaussian random effect in dim=k with Wishart prior

**hyper**

**theta1**

**hyperid** 29101

**name** theta1

**short.name** theta1

**initial** 1048576

**fixed** FALSE

**prior** wishartkd

**param** 100 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576  
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**to.theta** function(x) x

**from.theta** function(x) x

**theta2**

**hyperid** 29102

**name** theta2

**short.name** theta2

**initial** 1048576

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta3**

**hyperid** 29103

```

    name theta3
    short.name theta3
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 29104
    name theta4
    short.name theta4
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 29105
    name theta5
    short.name theta5
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 29106
    name theta6
    short.name theta6
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 29107
    name theta7
    short.name theta7
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 29108
    name theta8
    short.name theta8
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 29109
    name theta9
    short.name theta9
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 29110
    name theta10
    short.name theta10
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 29111
    name theta11
    short.name theta11
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta12
  hyperid 29112
  name theta12
  short.name theta12
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta13
  hyperid 29113
  name theta13
  short.name theta13
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta14
  hyperid 29114
  name theta14
  short.name theta14
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta15
  hyperid 29115
  name theta15
  short.name theta15
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta16
  hyperid 29116
  name theta16
  short.name theta16

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta17
    hyperid 29117
    name theta17
    short.name theta17
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta18
    hyperid 29118
    name theta18
    short.name theta18
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta19
    hyperid 29119
    name theta19
    short.name theta19
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta20
    hyperid 29120
    name theta20
    short.name theta20
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta21
  hyperid 29121
  name theta21
  short.name theta21
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta22
  hyperid 29122
  name theta22
  short.name theta22
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta23
  hyperid 29123
  name theta23
  short.name theta23
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta24
  hyperid 29124
  name theta24
  short.name theta24
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta25
  hyperid 29125

```

```

    name theta25
    short.name theta25
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta26
    hyperid 29126
    name theta26
    short.name theta26
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta27
    hyperid 29127
    name theta27
    short.name theta27
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta28
    hyperid 29128
    name theta28
    short.name theta28
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta29
    hyperid 29129
    name theta29
    short.name theta29
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta30
    hyperid 29130
    name theta30
    short.name theta30
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 29131
    name theta31
    short.name theta31
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 29132
    name theta32
    short.name theta32
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 29133
    name theta33
    short.name theta33
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

#### theta34

hyperid 29134  
name theta34  
short.name theta34  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta35

hyperid 29135  
name theta35  
short.name theta35  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta36

hyperid 29136  
name theta36  
short.name theta36  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta37

hyperid 29137  
name theta37  
short.name theta37  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta38

hyperid 29138  
name theta38  
short.name theta38

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta39
    hyperid 29139
    name theta39
    short.name theta39
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta40
    hyperid 29140
    name theta40
    short.name theta40
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta41
    hyperid 29141
    name theta41
    short.name theta41
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 29142
    name theta42
    short.name theta42
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta43
  hyperid 29143
  name theta43
  short.name theta43
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta44
  hyperid 29144
  name theta44
  short.name theta44
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta45
  hyperid 29145
  name theta45
  short.name theta45
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta46
  hyperid 29146
  name theta46
  short.name theta46
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta47
  hyperid 29147

```

```

    name theta47
    short.name theta47
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta48
    hyperid 29148
    name theta48
    short.name theta48
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta49
    hyperid 29149
    name theta49
    short.name theta49
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta50
    hyperid 29150
    name theta50
    short.name theta50
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 29151
    name theta51
    short.name theta51
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta52
    hyperid 29152
    name theta52
    short.name theta52
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta53
    hyperid 29153
    name theta53
    short.name theta53
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta54
    hyperid 29154
    name theta54
    short.name theta54
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta55
    hyperid 29155
    name theta55
    short.name theta55
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

constr FALSE
nrow.ncol FALSE
augmented TRUE
aug.factor 1
aug.constr 1 2 3 4 5 6 7 8 9 10
n.div.by -1
n.required TRUE
set.default.values TRUE
status experimental
pdf iidkd

```

## Example

Just simulate some data and estimate the parameters back. This is for **order=4**.

```

library(mvtnorm)

n <- 300
m <- 4
N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
  for (j in 1:m) {
    if (i != j) {
      Sigma[i, j] <- rho^abs(i-j) * sqrt(Sigma[i, i] * Sigma[j, j])
    }
  }
}

y <- c()
yy <- rmvnorm(n, sigma = Sigma)
for(i in 1:m) {
  y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
  ## set parameters using 'theta1'.
  ## these are the default parameters.
  hyper = list(theta1 = list(
    param = c(100, rep(1, m), rep(0, m*(m-1)/2))))) ,
  data = data.frame(i = 1:N, y),
  ## fix precision as we have exact observations
  control.family = list(hyper = list(
    prec = list(initial = 15, fixed = TRUE))),
  verbose = FALSE)

## this is how the internal parameters are defined

```

```

L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))

## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
                           mle = c(diag(LL), LL[lower.tri(LL)]),
                           inla = r$mode$theta))

## this gives a list of sampled matrices (stdev's and correlations)
xx <- inla.iidkd.sample(10^4, r, "i")
## compute the mean
qq <- matrix(rowMeans(matrix(unlist(xx), nrow = m^2)), m, m)

iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(diag(qq), qq[lower.tri(qq)]),
                           true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))

```