

PC prior for the range in PRW2

This is the PC prior for the (log-)range for the PRW2 latent model, and is only ment for that model.

Parametrization

This is the prior for the range for the PRW2 model

$$x_t = 2\gamma x_{t-1} - \gamma^2 x_{t-2} + \epsilon_t, \quad |\gamma| < 1$$

where ϵ_t is iid zero mean Gaussian noise. This process is made stationary with unit variance, and the range, the 'distance' or 'time' to small correlation, is set to be

$$\rho = \exp\left(-\frac{\sqrt{12}h}{r}\right)$$

where h is (typical) distance or time, 'step-size', between the knots, here $h = 1$. The correlation function of the process is found to be

$$\rho(h) = \left(1 + h \frac{1 - \gamma^2}{1 + \gamma^2}\right) \gamma^h, \quad h = 0, 1, 2, \dots$$

The PC prior is derived using the limiting case, $\gamma \rightarrow 1^-$, as the base model. The distance if found to be

$$d(\gamma) = \sqrt{2\text{KLD}} = \sqrt{3 - 4\rho(1) + \rho(2)} + o(1)$$

In distance scale, the PC-prior is exponential with rate λ ,

$$\pi(d) = \lambda \exp(-\lambda d), \quad \lambda > 0.$$

The calibration of λ , is done so that

$$\text{Prob}(r < r_0) = \alpha, \quad r_0 > 0, \quad 0 < \alpha < 1$$

for given r_0 and α . This prior is also applied to PRW2 for irregular locations derived from the corresponding limiting process in continous time.

Specification

This prior is specified as follows, as

```
hyper = list(range = list(prior = "pc.prw2.range",
                           param = c(r0, alpha, h, lambda))))}
```

where $r0 = r_0$, $\alpha = \alpha$, $h = h$ (the step-size), $\lambda = \lambda$ (the step-size).

Let the locations where the PRW2 process defined is in variable `loc`, then the default value used for r_0 , if initially r_0 is set to $r_0 \leq 0$, is

```
r0 <- diff(range(loc)) / 4
```

The default value used for α , if initially α is set to $\alpha \leq 0$, is $\alpha = 0.5$. This means the median for default prior for the range, is one quarter of the length for which the PRW2 model is defined.

The 'step-size' h is a typical distance between the locations where process is defined and is $h = 1$ in the definition above. The default value for this parameter, if initially h is set to $h \leq 0$, is

```
h <- diff(range(loc)) / (length(loc) - 1)
```

Note that this does not always make sense.

The λ parameter is computed from the calibration as defined above, if initially λ is set to $\lambda \leq 0$. If $\lambda > 0$, then this value of λ is used without computing the calibration, and the values of r_0 and α are silently ignored.

Note that r_0 and h is in real scale, ie in the scale of `loc`.

Default values are `param = c(0, 0, 0, 0)`.

Example

```
param <- c(10, 0.5, 1, 0)
r <- exp(seq(-3, 4, by = 0.01))
lty <- 1
plot(r, inla.prw2.drange(r, param), ylim=c(0, 0.1), type = "l", lty = lty)
for (h in 1/2^(0:20)) {
  param[3] <- h
  lty <- lty+1
  lines(r, inla.prw2.drange(r, param), lty = lty)
}
```

Notes

The details for this prior is available in the PhD thesis with link [HERE](#) (ADD LINK LATER).