

# More general Gaussian likelihoods: GGaussian and GGaussianS

## Parameterisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\tau(y - \mu)^2\right)$$

for continuously responses  $y$  where

$\mu$ : is the the mean

$\tau$ : is the precision

These likelihood families generalise the normal procedure. We allow for two linear predictors in the model. One needs to be “simple”, only consists of fixed effects, while the other one is general and defined via the formula.

For family=“ggaussian”, then

$$\text{link}(\mu) = \text{formula} \quad \text{link.simple}(\tau) = \text{simple}$$

while for family=“ggaussianS”, then

$$\text{link}(\tau) = \text{formula} \quad \text{link.simple}(\mu) = \text{simple}$$

The default link is the *identity* for the mean and the *log* for the precision. We will describe each model separately, as the specifications are a little different.

## Family *ggaussian*

### Link-function

The mean is given by the linear predictor  $\eta$  from the formula

$$\text{link}(\mu) = \eta$$

where the default is the *identity*-link. The precision is given as

$$\text{link.simple}\left(\frac{1}{s}\tau\right) = \beta_1 z_1 + \beta_2 z_2 + \dots + \beta_m z_m, \quad m \leq 10$$

where  $s$  is a **fixed** scaling (or often log-offset) and similar to the argument *scale* for family=“gaussian”. For the log-link, the precision is expressed as

$$\tau = s \times \exp(\beta_1 z_1 + \beta_2 z_2 + \dots + \beta_m z_m)$$

**Note:** there is no default intercept in the simple model, so it is common to set  $z_1 = 1$ .

## Hyperparameters

The hyperparameters in the model are  $\theta_1 = \beta_1, \dots, \theta_m = \beta_m$ , and the priors are defined on  $\theta_1, \dots, \theta_m$ .

**Note:** The default prior for  $\theta_1$  is similar to the default prior for the precision for the Gaussian family, please change if you do not want this feature.

## Specification

- `family="ggaussian"`
- Required arguments: `inla.mdata`-object that defines the response,  $s$  and covariates. The `inla.mdata` object is defined as

$$\text{inla.mdata}(y, s, z_1, \dots, z_m)$$

where each argument are vectors of the same length.

- **Note:** The scaling argument  $s$  **MUST** be given as it has no default value. Often we can just use  $s = 1$  as it will auto-expand.
- **Note:**  $m = 0$  is allowed, which means that  $\tau = s$  with with log link.simple.

## Hyperparameter spesification and default values

```
family="ggaussian"
```

```
doc Generalized Gaussian
```

```
hyper
```

```
  theta1
```

```
    hyperid 66501
    name beta1
    short.name beta1
    output.name beta1 for ggaussian observations
    output.name.intern beta1 for ggaussian observations
    initial 4
    fixed FALSE
    prior normal
    param 9.33 0.61
    to.theta function(x) x
    from.theta function(x) x
```

```
  theta2
```

```
    hyperid 66502
    name beta2
    short.name beta2
    output.name beta2 for ggaussian observations
    output.name.intern beta2 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
```

```
  theta3
```

```
    hyperid 66503
```

```

name beta3
short.name beta3
output.name beta3 for ggaussian observations
output.name.intern beta3 for ggaussian observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta4
  hyperid 66504
  name beta4
  short.name beta4
  output.name beta4 for ggaussian observations
  output.name.intern beta4 for ggaussian observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta5
  hyperid 66505
  name beta5
  short.name beta5
  output.name beta5 for ggaussian observations
  output.name.intern beta5 for ggaussian observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta6
  hyperid 66506
  name beta6
  short.name beta6
  output.name beta6 for ggaussian observations
  output.name.intern beta6 for ggaussian observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10

```

```

    to.theta function(x) x
    from.theta function(x) x
theta7
  hyperid 66507
  name beta7
  short.name beta7
  output.name beta7 for ggaussian observations
  output.name.intern beta7 for ggaussian observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta8
  hyperid 66508
  name beta8
  short.name beta8
  output.name beta8 for ggaussian observations
  output.name.intern beta8 for ggaussian observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta9
  hyperid 66509
  name beta9
  short.name beta9
  output.name beta9 for ggaussian observations
  output.name.intern beta9 for ggaussian observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 66510
  name beta10
  short.name beta10
  output.name beta10 for ggaussian observations
  output.name.intern beta10 for ggaussian observations

```

```

initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x

```

```
survival FALSE
```

```
discrete FALSE
```

```
link default identity
```

```
link.simple default log
```

```
pdf ggaussian
```

### Example

```

n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
eta <- 0.1 + 1.1 * x + 2.2 * xx

s <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)
eta.prec <- 1 + 0.55 * z + 1.1 * zz

y <- eta + 1/sqrt(s * exp(eta.prec)) * rnorm(n)
Y <- inla.mdata(y, s, 1, z, zz)
r <- inla(Y ~ 1 + x + xx,
          data = list(Y = Y, x = x, xx = xx, z = z, zz = zz, s = s),
          family = "ggaussian")
summary(r)

```

## Family *ggaussianS*

### Link-function

This is the swapped version. The mean is given by a simple linear predictor

$$\text{link.simple}(\mu) = \text{off} + \beta_1 z_1 + \beta_2 z_2 + \dots + \beta_m z_m, \quad m \leq 10$$

where “off” is a fixed offset and where `link.simple` is the *identity*-link by default. **Note:** there is no default intercept in the simple model, so it is common to set  $z_1 = 1$ . The precision is specified in the formula

$$\text{link}(\tau) = \text{formula}$$

using the log-link as default.

### Hyperparameters

The hyperparameters in the model are  $\theta_1 = \beta_1, \dots, \theta_m = \beta_m$ , and the priors are defined on  $\theta_1, \dots, \theta_m$ .

### Specification

- `family="ggaussianS"`
- Required arguments: **inla.mdata**-object that defines the response, the offset “off” and covariates. The **inla.mdata** object is defined as

$$\text{inla.mdata}(y, \text{off}, z_1, \dots, z_m)$$

where each argument are vectors of the same length.

- **Note:** The offset argument **MUST** be given as it has no default value. Often we can just use `off= 0` as it will auto-expand.
- **Note:**  $m = 0$  is allowed, which means that  $\mu = \text{off}$  with with identity `link.simple`.

### Hyperparameter spesification and default values

`family="ggaussianS"`

**doc** Generalized GaussianS

**hyper**

**theta1**

**hyperid** 66601

**name** beta1

**short.name** beta1

**output.name** beta1 for ggaussianS observations

**output.name.intern** beta1 for ggaussianS observations

**initial** 0

**fixed** FALSE

**prior** normal

**param** 0 0.001

**to.theta** function(x) x

**from.theta** function(x) x

#### theta2

hyperid 66602  
name beta2  
short.name beta2  
output.name beta2 for ggaussianS observations  
output.name.intern beta2 for ggaussianS observations  
initial 0  
fixed FALSE  
prior normal  
param 0 0.001  
to.theta function(x) x  
from.theta function(x) x

#### theta3

hyperid 66603  
name beta3  
short.name beta3  
output.name beta3 for ggaussianS observations  
output.name.intern beta3 for ggaussianS observations  
initial 0  
fixed FALSE  
prior normal  
param 0 0.001  
to.theta function(x) x  
from.theta function(x) x

#### theta4

hyperid 66604  
name beta4  
short.name beta4  
output.name beta4 for ggaussianS observations  
output.name.intern beta4 for ggaussianS observations  
initial 0  
fixed FALSE  
prior normal  
param 0 0.001  
to.theta function(x) x  
from.theta function(x) x

#### theta5

hyperid 66605  
name beta5  
short.name beta5  
output.name beta5 for ggaussianS observations  
output.name.intern beta5 for ggaussianS observations  
initial 0  
fixed FALSE

```

    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 66606
    name beta6
    short.name beta6
    output.name beta6 for ggaussianS observations
    output.name.intern beta6 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 66607
    name beta7
    short.name beta7
    output.name beta7 for ggaussianS observations
    output.name.intern beta7 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 66608
    name beta8
    short.name beta8
    output.name beta8 for ggaussianS observations
    output.name.intern beta8 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 66609
    name beta9
    short.name beta9

```



```

output.name beta9 for ggaussianS observations
output.name.intern beta9 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x
theta10
  hyperid 66610
  name beta10
  short.name beta10
  output.name beta10 for ggaussianS observations
  output.name.intern beta10 for ggaussianS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 0.001
  to.theta function(x) x
  from.theta function(x) x

```

survival FALSE

discrete FALSE

link default log

link.simple default identity

pdf ggaussian

### Example

```

n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
off <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)

mean <- off + 0.1 + 1.1 * z + 2.2 * zz
prec <- exp(1 + 0.55 * x + 1.1 * xx)

y <- mean + (1/sqrt(prec)) * rnorm(n)
Y <- inla.mdata(y, off, 1, z, zz)
r <- inla(Y ~ 1 + x + xx,
  data = list(Y = Y, off = off, x = x, xx = xx, z = z, zz = zz),
  family = "ggaussianS")
summary(r)

```