

## qLogLogistic likelihood

### Parametrisation

The LogLogistic distribution has cumulative distribution function

$$F_0(y) = \frac{1}{1 + \lambda y^{-\alpha}}, \quad y > 0$$

if `variant=0`, or

$$F_1(y) = \frac{1}{1 + (\lambda y)^{-\alpha}}, \quad y > 0$$

if `variant=1`, where

$\alpha > 0$  is a shape parameter, and

$\lambda > 0$  is a scale parameter.

The  $\lambda$  is defined implicitly through the quantile, as

$$F_0(y_q) = q, \quad \text{or} \quad F_1(y_q) = q, \quad 0 < q < 1$$

and the linear predictor is defined on  $y_q$ .

### Link-functions

The parameter  $\lambda$  is linked to the linear predictor, implicitly through

$$y_q = \exp(\eta)$$

### Hyperparameters

The  $\alpha$  parameter is represented as

$$\theta = \log \alpha$$

and the prior is defined on  $\theta$ .

### Specification

- family equals `qloglogistic` (regression) or `qloglogisticsurv` (survival)
- `variant=0` (default) or 1, choosing between parameterisation  $F_0$  or  $F_1$ .
- Required arguments:  $y$  (regression) or an `inla.surv`-object using `inla.surv()` (for survival data), and `quantile=q`.

### Hyperparameter specification and default values

#### Regression:

**doc** A quantile loglogistic likelihood

#### hyper

**theta**

**hyperid** 60011

**name** log alpha

```
short.name alpha
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

survival FALSE

discrete FALSE

link default log neglog

pdf qloglogistic

### Survival:

doc A quantile loglogistic likelihood (survival)

hyper

theta

```
hyperid 60021
name log alpha
short.name alpha
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

survival TRUE

discrete FALSE

link default log neglog

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### Example

In the following example we estimate the parameters in a simulated case

```
lam_loglogistic = function(yq, alpha, q, variant = 0)
{
  if (variant == 0) {
    lambda = yq^alpha * (1/q-1)
  } else if (variant == 1) {
    lambda = 1/yq * (1/(1/q-1))^(1/alpha)
  } else
    stop("ERR")
  return (lambda)
}
```

```
}
```

```
rloglogistic = function(n, lambda, alpha, variant=0)
{
  u = runif(n)
  if (variant == 0) {
    y = (lambda/(1.0/u - 1.0))^(1.0/alpha)
  } else if (variant == 1) {
    y = (1.0/(1.0/u - 1.0))^(1.0/alpha) / lambda
  } else {
    stop("ERROR")
  }
}
```

```
n = 500
```

```
alpha = 2.1
```

```
x = c(scale(runif(n)))
```

```
eta = 1.1+2.2*x
```

```
yq = exp(eta)
```

```
for(variant in 0:1) {
```

```
  for(q in c(0.2, 0.8)) {
```

```
    print(paste("variant=", variant, "quantile=", q))
```

```
    lambda = lam_loglogistic(yq, alpha, q, variant=variant)
```

```
    y = rloglogistic(n,
```

```
      lambda = lambda,
```

```
      alpha = alpha,
```

```
      variant = variant)
```

```
    formula = y ~ 1 + x
```

```
    rr=inla(formula,
```

```
      family ="qloglogistic",
```

```
      data=data.frame(y, x),
```

```
      control.family = list(list(variant = variant, control.link = list(quantile = q))
```

```
    print("REGRESSION")
```

```
    print(summary(rr))
```

```
    event = rep(1,n)
```

```
    formula=inla.surv(y,event) ~ 1 + x
```

```
    r=inla(formula,
```

```
      family ="qloglogisticsurv",
```

```
      data = list(y=y, event=event, x=x),
```

```
      control.family = list(list(variant = variant, control.link = list(quantile = q))
```

```
    print("SURVIVAL")
```

```
    print(summary(r))
```

```
  }
```

```
}
```

## Notes

- Loglogisticsurv model can be used for right censored, left censored, interval censored data. If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observatios, `time = time / max(time)` or similar.