

Probabilities for Weibull cure model

Event time is ∞ with probability ρ , and has the following Weibull distribution otherwise:

$$f(t; \mu_i, \nu) = \mu_i \nu t^{\nu-1} \exp[-\mu_i t^\nu].$$

Given that a subject reached time L without cancer incidence, the density function for their cancer incidence time T is

$$\begin{aligned} pr(T = t | T > L) &= [(1 - \rho)f(t; \mu_i, \nu)] \Big/ \left[\rho + (1 - \rho) \int_L^\infty f(u; \mu_i, \nu) du \right] \\ &= \frac{\mu_i \nu t^{\nu-1} \exp[-\mu_i t^\nu]}{[\rho / (1 - \rho) + \exp(-\mu_i L^\nu)]}. \end{aligned}$$

The probability that an individual has a cancer time greater than r (right censoring) given that they are cancer-free at age L is

$$\begin{aligned} pr(T > t | T > L) &= \frac{\rho + (1 - \rho) \int_t^\infty f(u; \mu_i, \nu) du}{\rho + (1 - \rho) \int_L^\infty f(u; \mu_i, \nu) du} \\ &= \frac{\rho + (1 - \rho) \exp(-\mu_i t^\nu)}{\rho + (1 - \rho) \exp(-\mu_i L^\nu)}. \end{aligned}$$

Left censoring.

$$\begin{aligned} pr(T < t | T > L) &= \frac{(1 - \rho) \int_L^t f(u; \mu_i, \nu) du}{\rho + (1 - \rho) \int_L^\infty f(u; \mu_i, \nu) du} \\ &= \frac{(1 - \rho) \exp[-\mu_i (t^\nu - L^\nu)]}{1 - (1 - \rho) \exp(-\mu_i L^\nu)}. \end{aligned}$$

interval censoring

$$\begin{aligned} pr(t_1 < T < t_2 | T > L) &= \frac{(1 - \rho) \int_{\max(L, t_1)}^{t_2} f(u; \mu_i, \nu) du}{\rho + (1 - \rho) \int_L^\infty f(u; \mu_i, \nu) du} \\ &= \frac{(1 - \rho) \exp[-\mu_i (t_2^\nu - \max(L, t_1)^\nu)]}{1 - (1 - \rho) \exp(-\mu_i L^\nu)}. \end{aligned}$$