

Bym2 model for spatial effects

Parametrization

This model is a reparameterisation of the BYM-model, which is a union of the besag model u^* and a iid model v^* , so that

$$x = \begin{pmatrix} v^* + u^* \\ u^* \end{pmatrix}$$

where both u^* and v^* has a precision (hyper-)parameter. The length of x is $2n$ if the length of u^* (and v^*) is n . The BYM2 model uses a different parameterisation of the hyperparameters where

$$x = \begin{pmatrix} \frac{1}{\sqrt{\tau}} (\sqrt{1-\phi} v + \sqrt{\phi} u) \\ u \end{pmatrix}$$

where both u and v are *standardised* to have (generalised) variance equal to one. The *marginal* precision is then τ and the proportion of the marginal variance explained by the spatial effect (u) is ϕ .

Hyperparameters

The hyperparameters are the marginal precision τ and the mixing parameter ϕ . The marginal precision τ is represented as

$$\theta_1 = \log(\tau)$$

and the mixing parameter as

$$\theta_2 = \log\left(\frac{\phi}{1-\phi}\right)$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The bym2 model is specified inside the `f()` function as

```
f(<whatever>, model="bym2", graph=<graph>,  
  hyper=<hyper>, adjust.for.con.comp = TRUE)
```

The neighbourhood structure of \mathbf{x} is passed to the program through the `graph` argument.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected component, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

Hyperparameter spesification and default values

doc The BYM-model with the PC priors

hyper

theta1

hyperid 11001

name log precision

short.name prec

prior pc.prec

```

    param 1 0.01
    initial 4
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 11002
    name logit phi
    short.name phi
    prior pc
    param 0.5 0.5
    initial -3
    fixed FALSE
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))

constr TRUE

nrow.ncol FALSE

augmented TRUE

aug.factor 2

aug.constr 2

n.div.by

n.required TRUE

set.default.values TRUE

status experimental

pdf bym2

```

Example

Details on the implementation

This gives some details of the implementation, which depends on the following variables

- nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.
- nc2** Number of connected components in the graph with size ≥ 2 .
- scale.model** The value of the logical flag, if the model should be scaled or not. (Default FALSE)
- adjust.for.con.comp** The value of the logical flag if the `constr=TRUE` option should be reinterpreted.

The case `(scale.model==FALSE && adjust.for.con.comp == FALSE)`

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on $(-\infty, \infty)$ before the constraint.

The case `(scale.model==TRUE && adjust.for.con.comp == FALSE)`

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Let $Q = \tau R$ be the standard precision matrix from the `besag`-model with precision parameter τ . Then R , except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and R is modified so that singletons have a standard Normal distribution.

The case `(scale.model==FALSE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as one sum-to-zero constraint over each of the `nc2` connected components of size ≥ 2 . Singletons are given a uniform distribution on $(-\infty, \infty)$.

The case `(scale.model==TRUE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as `nc2` sum-to-zero constraints for each of the connected components of size ≥ 2 . Let $Q = \tau R$ be the standard precision matrix from the `besag`-model with precision parameter τ . Then R , are scaled so that the geometric mean of the marginal variances in each connected component of size ≥ 2 is 1, and modified so that singletons have a standard Normal distribution.

Notes

The term $\frac{1}{2} \log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here R is the precision matrix for the standardised Besag part of the model.

The generic PC-prior for ϕ is available as `prior="pc"` and parameters `param="c(u, alpha)",` where $\text{Prob}(\phi \leq u) = \alpha$. If $\alpha < 0$ or $\alpha > 1$, then it is set to a value close to the minimum value of α allowed. This prior depends on the graph and its computational cost is $\mathcal{O}(n^3)$.