Autoregressive model of order 1 (AR1)

Parametrization

The autoregressive model of order 1 (AR1) for the Gaussian vector $\mathbf{x} = (x_1, \ldots, x_n)$ is defined as:

$$
\begin{aligned}
x_1 &\sim \mathcal{N}(0, (\tau(1 - \rho^2))^{-1}) \\
x_i &= \rho\, x_{i-1} + \epsilon_i; \qquad \epsilon_i \sim \mathcal{N}(0, \tau^{-1}) \qquad i = 2, \ldots, n
\end{aligned}
$$

where

$$|\rho| < 1$$

Hyperparameters

The precision parameter κ is represented as

$$\theta_1 = \log(\kappa)$$

where κ is the *marginal* precision,

$$\kappa = \tau(1 - \rho^2).$$

The parameter ρ is represented as

$$\theta_2 = \log\left(\frac{1 + \rho}{1 - \rho}\right)$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The AR1 model is specified inside the `f()` function as

```
 f(<whatever>, model="ar1", values=<values>, hyper = <hyper>)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See the example for RW1 for an application.

Hyperparameter spesification and default values

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hyper

    theta1

        hyperid 14001
        name log precision
        short.name prec
        prior loggamma
        param 1 5e-05
        initial 4
        fixed FALSE
        to.theta `function(x) log(x)`
        from.theta `function(x) exp(x)`

    theta2

        hyperid 14002

name logit lag one correlation

short.name rho

prior normal

param 0 0.15

initial 2

fixed FALSE

to.theta `function(x) log((1 + x) / (1 - x))`

from.theta `function(x) 2 * exp(x) / (1 + exp(x)) - 1`

theta3

hyperid 14003

name mean

short.name mean

prior normal

param 0 1

initial 0

fixed TRUE

to.theta `function(x) x`

from.theta `function(x) x`

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf ar1

Example

In this exaple we implement an ar1 model observed with Poisson counts

```
#simulate data
n = 100
rho = 0.8
prec = 10
## note that the marginal precision would be
marg.prec = prec * (1-rho^2)

E=sample(c(5,4,10,12),size=n,replace=T)
eta = as.vector(arima.sim(list(order = c(1,0,0), ar = rho), n = n,sd=sqrt(1/prec)))
y=rpois(n,E*exp(eta))
data = list(y=y, z=1:n, E=E)
```

```
## fit the model
formula = y~f(z,model="ar1")
result = inla(formula,family="poisson", data = data, E=E)
```

Notes

A third hyperparameter θ_3 is **experimental**, and the *mean* of the AR1 process. By default this parameter is fixed to the be zero.