Generalized Pareto distribution

Parametrisation

The generalized Pareto (GP) distribution with positive shape parameter has cumulative distribution function

$$F(y;\sigma,\xi) = 1 - \left(1 + \xi\frac{y}{\sigma}\right)^{-1/\xi}, \quad y > 0,$$

for a continuous response y where

ξ: is the tail parameter, $\xi > 0$

σ: is the scale parameter, $\sigma > 0$

The limit for $\xi \downarrow 0$ is $F(y;\sigma,0) = 1 - \exp(-x/\sigma)$.

Link function

The linear predictor η controls the α quantile of the GP

$$P(y \le q_\alpha) = \alpha$$

and $q_\alpha = \exp(\eta)$. The scaling σ, is then a function of (q_α, ξ), as

$$\sigma = \frac{\xi \exp(\eta)}{(1-\alpha)^{-\xi} - 1}$$

Hyperparameters

The GP model has one hyperparameter. The tail $\xi > 0$ is represented as

$$\xi = \xi_{\text{low}} + (\xi_{\text{high}} - \xi_{\text{low}})\frac{\exp(\theta)}{1+\exp(\theta)}$$

and the prior is defined on θ, with constant low and high values. The prior is FIXED to `pc.gevtail`, see `inla.doc("pc.gevtail")` for more info.

Specification

- family=`gp`
- Required arguments: y and the quantile α.

The quantile is given as `control.family=list(control.link=list(quantile=`α`))`.

Hyperparameter spesification and default values

doc Generalized Pareto likelihood

hyper

theta

hyperid 101201
name tail
short.name xi
initial -4
fixed FALSE
prior pc.gevtail
param 7 0 0.5
to.theta `function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) log(-(interva`
from.theta `function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) interval[1]`

status experimental

survival FALSE

discrete TRUE

link default quantile

pdf genPareto

Example

```
rgp = function(n, sigma, eta, alpha, xi = 0.001)
{
    if (missing(sigma)) {
        stopifnot(!missing(eta) && !missing(alpha))
        sigma = exp(eta) * xi / ((1.0 - alpha)^(-xi) -1.0)
    }
    return (sigma / xi * (runif(n)^(-xi) -1.0))
}

n = 300
x = runif(n)-0.5
eta = 1+x
alpha = 0.95
xi = 0.3
y = rgp(n, eta = eta, alpha = alpha, xi=xi)

r = inla(y ~ 1+x,
```

```
          data = data.frame(y, x),
          family = "gp",
          control.family = list(control.link = list(quantile = alpha)),
          control.predictor = list(compute=TRUE),
          verbose=TRUE)


rx = range(c(r$summary.fitted.values$mean, exp(eta)))
plot(r$summary.fitted.values$mean, exp(eta),
     xlim = rx, ylim = rx)
abline(a=0,b=1)
```

Notes

The prior for ξ is fixed to `pc.gevtail` or one that respect an given interval similarly. Note that the default prior (and the internal representation of it and its name) changed 2019/10/12.