

Likelihood fmri

Parametrisation

This is special parameterisation of the non-central χ_ν distribution. Let $\{x_i\}$ are iid Gaussians with mean μ and precision τ , then

$$z = \sqrt{\sum_{i=1}^{\nu} \tau x_i^2}$$

is non-central χ -distribution with (integer and fixed by design) $\nu > 0$ degrees of freedom, and non-centrality parameter

$$\rho = \sqrt{\nu\tau\mu^2}.$$

The observation y is $y = z/\sqrt{\tau}$ and we're interested in the underlying true signal

$$\lambda = \rho/\sqrt{\tau} = \sqrt{\nu\mu^2}$$

Link-function

The linkfunction is given as

$$\log(\lambda) = \eta$$

where η is the linear predictor.

Hyperparameters

The hyperparameters are $\theta = (\theta_1, \theta_2)$, where

$$\tau = \exp(\theta_1)$$

is the precision, and

$$\nu = \theta_2$$

For technical reasons, ν is implemented as a hyper-parameter, but is required to be fixed. Hence, the initial value for θ_2 defines the (fixed) value for ν .

The prior is given on θ_1 .

Specification

- family = `fmri` or family = `fmrismrv`
- Required arguments: y (and optional `scale` for `fmri` to scale τ)

Hyperparameter specification and default values

`doc` fmri distribution (special nc-chi)

`hyper`

`theta1`

`hyperid` 103101

`name` precision

`short.name` prec

`initial` 0

`fixed` FALSE

```
prior loggamma
param 10 10
to.theta function(x) log(x)
from.theta function(x) exp(x)
theta2
  hyperid 103202
  name dof
  short.name df
  initial 4
  fixed TRUE
  prior normal
  param 0 1
  to.theta function(x) x
  from.theta function(x) x

status experimental

survival FALSE

discrete FALSE

link default log

pdf fmri

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hyper
```

```
theta1
  hyperid 104101
  name precision
  short.name prec
  initial 0
  fixed FALSE
  prior loggamma
  param 10 10
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
```

```
theta2
  hyperid 104201
  name dof
  short.name df
  initial 4
  fixed TRUE
  prior normal
  param 0 1
  to.theta function(x) x
```

```
from.theta function(x) x
```

```
status experimental
```

```
survival TRUE
```

```
discrete FALSE
```

```
link default log
```

```
pdf fmri
```

Example

In the following example we estimate the parameters in a simulated example.

```
n <- 300
x <- rnorm(n, sd = 0.3)
df <- 1
prec <- 3
eta <- 1 + x
lambda <- exp(eta)
y <- sqrt(rchisq(n, df = df, ncp = prec * lambda^2) /prec)

r <- inla(y ~ 1 + x,
         data = data.frame(y, x),
         family = "fmri",
         control.family = list(hyper = list(df = list(initial = df))),
         control.inla = list(cmin = 0,
                             int.strategy = "eb",
                             strategy = "adaptive"),
         verbose = TRUE)
summary(r)

## 'cmin=0' seems to be required only for initial values that can give
## 'crazy' values. We can rerun without this re-starting at the prev fit,
## to validate
r$.args$control.inla$cmin <- -Inf
r$.args$control.inla$int.strategy <- "auto"
rr <- inla.rerun(r)
summary(rr)
```

Notes

Thanks to LS for providing all the details and a robust implementation of this likelihood.