

## Besag model for spatial effects

### Parametrization

The besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i | x_j, i \neq j, \tau \sim \mathcal{N}\left(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau}\right) \quad (1)$$

where  $n_i$  is the number of neighbours of node  $i$ ,  $i \sim j$  indicates that the two nodes  $i$  and  $j$  are neighbours.

### Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

### Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besag", graph=<graph>,
  hyper=<hyper>, adjust.for.con.comp = TRUE,
  scale.model = FALSE)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph` argument.

If the option `adjust.for.con.comp=TRUE` then the model is adjusted if the graph has more than one connected component. This adjustment can be disabled setting this option to `FALSE`. If `adjust.for.con.comp=TRUE` then `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component in the graph and the `rankdef` parameter is set to the number of connected components.

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior specification much easier. Default is `FALSE` so that the model is not scaled.

### Hyperparameter spesification and default values

**doc** The Besag area model (CAR-model)

**hyper**

**theta**

**hyperid** 8001

**name** log precision

**short.name** prec

**prior** loggamma

**param** 1 5e-05

**initial** 4

**fixed** FALSE

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

**constr** TRUE  
**nrow.ncol** FALSE  
**augmented** FALSE  
**aug.factor** 1  
**aug.constr**  
**n.div.by**  
**n.required** TRUE  
**set.default.values** TRUE  
**pdf** besag

### Example

For examples of application of this model see the `Bym`, `Munich`, `Zambia` or `Scotland` examples in Volume I.

## Details on the implementation

This gives some details of the implementation, which depends on the following variables

**nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

**nc2** Number of connected components in the graph with size  $\geq 2$ .

**scale.model** The value of the logical flag, if the model should be scaled or not. (Default FALSE)

**adjust.for.con.comp** The value of the logical flag if the `constr=TRUE` option should be reinterpreted.

**The case** (`scale.model==FALSE && adjust.for.con.comp == FALSE`)

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on  $(-\infty, \infty)$  before the constraint.

**The case** (`scale.model==TRUE && adjust.for.con.comp == FALSE`)

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Let  $Q = \tau R$  be the standard precision matrix from the `besag`-model with precision parameter  $\tau$ . Then  $R$ , except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and  $R$  is modified so that singletons have a standard Normal distribution.

**The case** (`scale.model==FALSE && adjust.for.con.comp == TRUE`)

The option `constr=TRUE` is interpreted as one sum-to-zero constraint over each of the `nc2` connected components of size  $\geq 2$ . Singletons are given a uniform distribution on  $(-\infty, \infty)$ .

**The case** `(scale.model==TRUE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as `nc2` sum-to-zero constraints for each of the connected components of size  $\geq 2$ . Let  $Q = \tau R$  be the standard precision matrix from the `besag`-model with precision parameter  $\tau$ . Then  $R$ , are scaled so that the geometric mean of the marginal variances in each connected component of size  $\geq 2$  is 1, and modified so that singletons have a standard Normal distribution.

## Notes

The term  $\frac{1}{2} \log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here  $R$  is the precision matrix with a unit precision parameter.