

The Kumaraswamy distribution

Parametrisation

The Kumaraswamy distribution is

$$f(y) = \alpha\beta y^{\alpha-1}(1-y^\alpha)^{\beta-1}$$

for $0 < y < 1$ and $\alpha, \beta > 0$. The cumulative distribution function is

$$F(y) = 1 - (1 - y^\alpha)^\beta.$$

The parametrisation is given in terms of the quantile function

$$\kappa(q) = \left(1 - (1 - q)^{1/\beta}\right)^{1/\alpha}$$

and the precision parameter ϕ ,

$$\phi(q) = -\ln\left(1 - (1 - q)^{1/\beta}\right)$$

for *fixed* value of $0 < q < 1$.

Link-function

The quantile κ to the linear predictor by

$$\text{logit}(\kappa) = \eta$$

using the default logit link-function.

Hyperparameters

The hyperparameter is

$$\phi = \exp(S\theta)$$

and the prior is defined on θ . The constant S currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of α can make a huge difference.

Specification

- family = qkumar
- Required arguments: y and the quantile q .

The quantile is given as `control.family=list(control.link = list(quantile=q))`.

Hyperparameter specification and default values

doc A quantile version of the Kumar likelihood

hyper

theta

hyperid 60001

name precision parameter

short.name prec

initial 1

```
fixed FALSE
prior loggamma
param 1 0.1
to.theta function(x, sc = 0.1) log(x) / sc
from.theta function(x, sc = 0.1) exp(sc * x)
```

survival FALSE

discrete FALSE

link default logit loga cauchit

pdf qkumar

Example

```
rkumar = function(n, eta, phi, q=0.5)
{
  kappa = eta
  beta = log(1-q)/log(1-exp(-phi))
  alpha = log(1- (1-q)^(1/beta)) / log(kappa)
  u = runif(n)
  y = (1-u^(1/beta))^(1/alpha)
  return (y)
}

n = 100
q = 0.5
phi = 1
x = rnorm(n, sd = 1)
eta = inla.link.invlogit(1 + x)
y = rkumar(n, eta, phi, q)
r = inla(y ~ 1 + x,
  data = data.frame(y, x),
  family = "qkumar",
  control.family = list(control.link=list(quantile = q)))
summary(r)
```

Notes

None.