Special/uncommon versions of the Poisson

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!} \exp(-\lambda)$$

for responses $y = 0, 1, 2, \ldots$, where

λ: the expected value.

Various special versions can be defined from here.

Special1

Here we consider a 1-inflated modification where there its known there are no zeros

$$\text{Prob}(y) = p \times 1_{[y=1]} + \frac{1-p}{1-\exp(-\lambda)} \times \frac{\lambda^y}{y!} \exp(-\lambda), \qquad y = 1, 2, \ldots$$

Link-function

λ is linked to the linear predictor by

$$\lambda = E \exp(\eta)$$

where $E > 0$ is a known constant (or $\log(E)$ is the offset of η).

Hyperparameters

Special1 have hyperparmeter p which is represented as

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior is given on θ

doc The Poisson.special1 likelihood

hyper

- **theta**
  - **hyperid** 56100
  - **name** logit probability
  - **short.name** prob
  - **initial** -1
  - **fixed** FALSE
  - **prior** gaussian
  - **param** -1 0.2
  - **to.theta** `function(x) log(x / (1 - x))`
  - **from.theta** `function(x) exp(x) / (1 + exp(x))`

survival FALSE

discrete TRUE

link default log

pdf poisson-special

Specification

- family = `poisson.special1`
- Required arguments: (integer-valued) y and E

Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n <- 300
a <- 1
b <- 1
p <- 0.2
x <- rnorm(n, sd = 0.2)
mu <- exp(a+b*x)
y.max <- ceiling(max(mu + 10*sqrt(mu)))
y <- numeric(n)

for(i in 1:n) {
    yy <- 1:y.max
    dy <- dpois(yy, lambda = mu[i])
    dy <- dy/sum(dy)
    dy <- dy * (1-p)
    dy[1] <- dy[1] + p
    y[i] <- sample(x = yy, size = 1, prob = dy)
}

r <- inla(y ~ 1 + x,
          data = data.frame(y, x),
          family = "poisson.special1")
summary(r)
```

Notes