

# Bym model for spatial effects

## Parametrization

This model is simply a union of the besag model  $u$  and a iid model  $v$ , so that

$$x = \begin{pmatrix} v + u \\ u \end{pmatrix}$$

Note that the length of  $x$  is  $2n$  if the length of  $u$  (and  $v$ ) is  $n$ . The benefite is that this allows to get the posterior marginals of the sum of the spatial and iid model; otherwise it offers no advantages.

## Hyperparameters

The hyperparameters are the precision  $\tau_1$  of the iid model ( $v$ ) and the precision  $\tau_2$  of the besag model ( $u$ ). The precision parameters are represented as

$$\theta = (\theta_1, \theta_2) = (\log \tau_1, \log \tau_2)$$

and the prior is defined on  $\theta$ .

## Specification

The bym model is specified inside the `f()` function as

```
f(<whatever>, model="bym", graph=<graph>,  
  hyper=<hyper>, adjust.for.con.comp = TRUE,  
  scale.model = FALSE)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph` argument.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected compoment, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

The logical option `scale.model` determine if the besag-model-part of the model  $u$  should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

## Hyperparameter spesification and default values

**doc** The BYM-model (Besag-York-Mollier model)

**hyper**

**theta1**

```
hyperid 10001  
name log unstructured precision  
short.name prec.unstruct  
prior loggamma  
param 1 5e-04  
initial 4  
fixed FALSE  
to.theta function(x) log(x)  
from.theta function(x) exp(x)
```

```

theta2
  hyperid 10002
  name log spatial precision
  short.name prec.spatial
  prior loggamma
  param 1 5e-04
  initial 4
  fixed FALSE
  to.theta function(x) log(x)
  from.theta function(x) exp(x)

constr TRUE

nrow.ncol FALSE

augmented TRUE

aug.factor 2

aug.constr 2

n.div.by

n.required TRUE

set.default.values TRUE

pdf bym

```

## Example

For examples of application of this model see the `Bym` example in Volume I.

## Details on the implementation

This gives some details of the implementation, which depends on the following variables

**nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

**nc2** Number of connected components in the graph with size  $\geq 2$ .

**scale.model** The value of the logical flag, if the model should be scaled or not. (Default FALSE)

**adjust.for.con.comp** The value of the logical flag if the `constr=TRUE` option should be reinterpreted.

**The case** (`scale.model==FALSE && adjust.for.con.comp == FALSE`)

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on  $(-\infty, \infty)$  before the constraint.

**The case (scale.model==TRUE && adjust.for.con.comp == FALSE)**

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Let  $Q = \tau R$  be the standard precision matrix from the `besag`-model with precision parameter  $\tau$ . Then  $R$ , except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and  $R$  is modified so that singletons have a standard Normal distribution.

**The case (scale.model==FALSE && adjust.for.con.comp == TRUE)**

The option `constr=TRUE` is interpreted as one sum-to-zero constraint over each of the `nc2` connected components of size  $\geq 2$ . Singletons are given a uniform distribution on  $(-\infty, \infty)$ .

**The case (scale.model==TRUE && adjust.for.con.comp == TRUE)**

The option `constr=TRUE` is interpreted as `nc2` sum-to-zero constraints for each of the connected components of size  $\geq 2$ . Let  $Q = \tau R$  be the standard precision matrix from the `besag`-model with precision parameter  $\tau$ . Then  $R$ , are scaled so that the geometric mean of the marginal variances in each connected component of size  $\geq 2$  is 1, and modified so that singletons have a standard Normal distribution.

## Notes

The term  $\frac{1}{2} \log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here  $R$  is the precision matrix with a unit precision parameter for the Besag part of the model.