

# The Gamma-distribution

## Parametrisation

The Gamma-distribution has the following density

$$\pi(y) = \frac{b^a}{\Gamma(a)} y^{a-1} \exp(-by), \quad a > 0, \quad b > 0, \quad y > 0,$$

where  $E(y) = \mu = a/b$  and  $\text{Var}(y) = 1/\tau = a/b^2$ , where  $\tau$  is the precision and  $\mu$  is the mean. We will use the following parameterisation for the precision

$$\tau = (s\phi)/\mu^2$$

where  $\phi$  is the precision parameter (or  $1/\phi$  is the dispersion parameter) and  $s > 0$  is a fixed scaling (for the regression model), which gives this density

$$\pi(y) = \frac{1}{\Gamma(s\phi)} \left( \frac{(s\phi)}{\mu} \right)^{(s\phi)} y^{(s\phi)-1} \exp \left( - (s\phi) \frac{y}{\mu} \right)$$

## Link-function

The linear predictor  $\eta$  is linked to the mean  $\mu$  using a default log-link

$$\mu = \exp(\eta)$$

## Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = \exp(\theta)$$

and the prior is defined on  $\theta$ .

## Specification

- `family="gamma"` for regression models and `family="gamma.surv"` for survival models.
- Required arguments: for `gamma.surv`,  $y$  (to be given in a format by using `inla.surv()`), and for `gamma`,  $y$  and  $s$  (default value 1).

The scalings  $s$  is **not** used for `family="gamma.surv"`.

## Hyperparameter spesification and default values

`doc` The Gamma likelihood

`hyper`

`theta`

`hyperid` 58001

`name` precision parameter

`short.name` prec

`output.name` Precision-parameter for the Gamma observations

`output.name.intern` Intern precision-parameter for the Gamma observations

```

    initial 4.60517018598809
    fixed FALSE
    prior loggamma
    param 1 0.01
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log quantile

pdf gamma

doc The Gamma likelihood (survival)

hyper
  theta1
    hyperid 58101
    name precision parameter
    short.name prec
    output.name Precision-parameter for the Gamma surv observations
    output.name.intern Intern precision-parameter for the Gamma surv observations
    initial 0
    fixed FALSE
    prior loggamma
    param 1 0.01
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
  theta2
    hyperid 58102
    name beta1
    short.name beta1
    output.name beta1 for Gamma-Cure
    output.name.intern beta1 for Gamma-Cure
    initial -7
    fixed FALSE
    prior normal
    param -4 100
    to.theta function(x) x
    from.theta function(x) x
  theta3
    hyperid 58103
    name beta2
    short.name beta2
    output.name beta2 for Gamma-Cure

```

```

    output.name.intern beta2 for Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 58104
    name beta3
    short.name beta3
    output.name beta3 for Gamma-Cure
    output.name.intern beta3 for Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 58105
    name beta4
    short.name beta4
    output.name beta4 for Gamma-Cure
    output.name.intern beta4 for Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 58106
    name beta5
    short.name beta5
    output.name beta5 for Gamma-Cure
    output.name.intern beta5 for Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7

```

```

hyperid 58107
name beta6
short.name beta6
output.name beta6 for Gamma-Cure
output.name.intern beta6 for Gamma-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta8
hyperid 58108
name beta7
short.name beta7
output.name beta7 for Gamma-Cure
output.name.intern beta7 for Gamma-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta9
hyperid 58109
name beta8
short.name beta8
output.name beta8 for Gamma-Cure
output.name.intern beta8 for Gamma-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta10
hyperid 58110
name beta9
short.name beta9
output.name beta9 for Gamma-Cure
output.name.intern beta9 for Gamma-Cure
initial 0
fixed FALSE
prior normal

```

```

    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 58111
    name beta10
    short.name beta10
    output.name beta10 for Gamma-Cure
    output.name.intern beta10 for Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

survival TRUE

discrete FALSE

status experimental

link default log neglog quantile

pdf gammasurv

```

## Example

In the following example we estimate the parameters in a simulated example.

```

n = 1000
x = rnorm(n)
eta = 1 + x
mu = exp(eta)
prec.scale = runif(n, min = 0.5, max = 2)
prec.par = 1.2
a = prec.par * prec.scale
b = mu / (prec.par * prec.scale)
y = rgamma(n, shape = a, scale = b)
r = inla(y ~ 1 + x, data = data.frame(y, x),
        scale = prec.scale, family = "gamma")

```

## Notes

None.