

# Autoregressive model of order 1 with covariates (AR1C)

## Parametrization

This is an extension of the common autoregressive model (AR1) to include a set of covariates into the conditional mean

$$\begin{aligned}x_1 &\sim \mathcal{N}(0, (\tau(1 - \rho^2))^{-1}) \\x_t &= \rho x_{t-1} + \sum_{j=1}^m \beta_j z_{t-1}^{(j)} + \epsilon_t; \quad \epsilon_i \sim \mathcal{N}(0, \tau^{-1}) \quad t = 2, \dots, n\end{aligned}$$

where  $|\rho| < 1$ . The latent vector has length  $n + m$  and is represented as  $(x_1, x_2, \dots, x_n, \beta_1, \dots, \beta_m)$ .

## Hyperparameters

The precision parameter  $\kappa$  is represented as

$$\theta_1 = \log(\kappa)$$

where  $\kappa$  is the *marginal* precision (when there is no covariates)

$$\kappa = \tau(1 - \rho^2).$$

The parameter  $\rho$  is represented as

$$\theta_2 = \log\left(\frac{1 + \rho}{1 - \rho}\right)$$

and the prior is defined on  $\theta = (\theta_1, \theta_2)$ .

## Specification

The AR1C model is specified as

```
f(<whatever>, model="ar1c", hyper = <hyper>,
  args.ar1c = list(Z=Z, Q.beta = Q))
```

The covariates are given in the **matrix**  $Z$  and **must have** dimension  $n \times m^1$ . The prior for  $\beta = (\beta_1, \dots, \beta_m)$  is a zero mean Gaussian with a  $m \times m$  precision **matrix**  $Q$ .

## Hyperparameter specification and default values

**doc** Auto-regressive model of order 1 w/covariates

**hyper**

**theta1**

**hyperid** 14101

**name** log precision

**short.name** prec

**prior** pc.prec

**param** 1 0.01

**initial** 4

**fixed** FALSE

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<sup>1</sup>Despite the fact that the last row of  $Z$  is not used

```

    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
  hyperid 14102
  name logit lag one correlation
  short.name rho
  prior pc.cor0
  param 0.5 0.5
  initial 2
  fixed FALSE
  to.theta function(x) log((1 + x) / (1 - x))
  from.theta function(x) 2 * exp(x) / (1 + exp(x)) - 1

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values TRUE

pdf ar1c

```

## Example

```

n = 500
N = n+1
phi = 0.98
sd.h = 0.4
prec = (1/sd.h)^2
prec.prime = prec / (1-phi^2)
beta = 0.05
h = numeric(n)
y = numeric(n)
z = numeric(n)
s = 0.01
z[n] = NA ## not used
h[1] = rnorm(1, sd = sqrt(1/prec))
y[1] = rnorm(1, sd = sd.h) + rnorm(1, sd = s)
for(i in 2:n) {
  z[i-1] = rnorm(1)
  h[i] = phi * h[i-1] + beta * z[i-1] +
    rnorm(1, sd = sqrt(1/prec.prime))
  y[i] = h[i] + rnorm(1, sd = s)
}

```

```

idx = 1:n

r = inla(y ~ -1 + f(idx, model="ar1c",
               args.ar1c = list(Z = cbind(z),
                                Q.beta = matrix(1, 1, 1))),
        data = data.frame(y, idx),
        family = "gaussian",
        control.family = list(
          hyper = list(prec = list(
            initial = log(1/s^2),
            fixed=TRUE))))

par(mfrow=c(2, 2))
plot(idx, y, type="l", main = "data")
plot(inla.tmarginal(function(x) sqrt(1/exp(x)),
                    r$internal.marginals.hyperpar[[1]]),
      type = "l", lwd=2, main = "sd(h)")
abline(v = sd.h)

plot(inla.smarginal(r$marginals.hyperpar[[2]]),
      type = "l", lwd=2, main = "phi")
abline(v = phi)

## the N'th element is 'beta'
plot(inla.smarginal(r$marginals.random$idx[[N]]), type="l", lwd=2,
      main = "beta")
abline(v = beta)

```

## Notes

- If  $m = 0$ , use `model="ar1"`.