

# Extended Generalised Pareto Distribution (EGP)

## Parametrisation

The cummulative distribution function for the EGP distribution, is

$$F(y) = \left(1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}\right)^\kappa, \quad y > 0.$$

Here,

- $\xi$  is the tail-parameter,
- $\sigma > 0$  is the scale-parameter, and
- $\kappa > 0$  is the shape-parameter.

## Link-function

The linear predictor  $\eta$  controls the  $\alpha$ -quantile of EGP

$$\text{Prob}(y \leq q_\alpha) = \alpha.$$

$\sigma$  can be expressed in terms of the quantile

$$\sigma = \frac{\xi q_\alpha}{(1 - \alpha^{1/\kappa})^{-\xi} - 1}$$

and the quantile is linked to the linear predictor as

$$q_\alpha = \exp(\eta).$$

## Hyperparameters

There are two hyperparameters, the tail and shape parameter, represented as  $(\theta_1, \theta_2)$ . The tail-parameter is in a predefined range  $(\xi_L, \xi_H)$

$$\xi = \xi_L + (\xi_H - \xi_L) \frac{1}{1 + \exp(-\theta_1)}$$

and the prior is defined on  $\theta_1$ . The shape-parameter is

$$\kappa = \exp(\theta_2)$$

and the prior is defined on  $\theta_2$ .

## Specification

- `family="egp"`
- Required arguments: the observations  $y$
- The quantile level  $\alpha$

## Hyperparameter spesification and default values

doc Extended Generalized Pareto likelihood

hyper

theta1

hyperid 101211

name tail

short.name xi

output.name Tail parameter for egp observations

output.name.intern Intern tail parameter for egp observations

initial 0

fixed FALSE

prior pc.egptail

param 5 -0.5 0.5

to.theta function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) log(-(interval[1] - x) /

from.theta function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) interval[1] + (interval

theta2

hyperid 101212

name shape

short.name kappa

output.name Shape parameter for the egp observations

output.name.intern Intern shape parameter for the egp observations

initial 0

fixed FALSE

prior loggamma

param 100 100

to.theta function(x) log(x)

from.theta function(x) exp(x)

status experimental

survival FALSE

discrete FALSE

link default quantile

pdf egp

## Example

```
n <- 3000
for (xi in c(-0.3, 0.3)) {
  xi.intern <- inla.models()$likelihood$egp$hyper$theta1$to.theta(xi, interval=c(-0.5, 0.5))
  alpha <- 0.9
  kappa <- 1.1
  kappa.intern <- inla.models()$likelihood$egp$hyper$theta2$to.theta(kappa)
  a <- ((1-alpha^(1/kappa))^-xi) -1)
  x <- rnorm(n, sd = 0.3)
  eta <- 0.9 + 1.1 * x
  q <- exp(eta)
  sigma <- xi * q / a
  y <- - (sigma / xi) * (1- (1-runif(n)^(1/kappa))^-xi))

  r <- inla(y ~ 1 + x,
    family = "egp",
    control.family = list(
      control.link = list(quantile = alpha),
      hyper = list(tail = list(
        ##initial = xi.intern,
        fixed = !TRUE,
        prior = "pc.egptail",
        param = c(5, -0.5, 0.5)),
      shape = list(
        ##initial = kappa.intern,
        fixed = !TRUE,
        prior = "loggamma",
        param = c(100, 100))))),
    data = data.frame(y, x),
    verbose = !TRUE)
  print(summary(r))
}
```