

# The Beta-distribution

## Parametrisation

The Beta-distribution has the following density

$$\pi(y) = \frac{1}{B(a, b)} y^{a-1} (1-y)^{b-1}, \quad 0 < y < 1, \quad a > 0, \quad b > 0$$

where  $B(a, b)$  is the Beta-function

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and  $\Gamma(x)$  is the Gamma-function. The (re-)parameterisation used is

$$\mu = \frac{a}{a+b}, \quad 0 < \mu < 1$$

and

$$\phi = a+b, \quad \phi > 0,$$

as it makes

$$E(y) = \mu \quad \text{and} \quad \text{Var}(y) = \frac{\mu(1-\mu)}{1+\phi}.$$

The parameter  $\phi$  is known as the *precision parameter*, since for fixed  $\mu$ , the larger  $\phi$  the smaller the variance of  $y$ . The parameters  $\{a, b\}$  are given as  $\{\mu, \phi\}$  as follows,

$$a = \mu\phi \quad \text{and} \quad b = -\mu\phi + \phi.$$

In some applications then observations close to 0 or 1, are censored and represented as exactly 0 and 1. For this, we introduced a censor value  $0 < \delta < 1/2$  and treat all  $y \leq \delta$  or  $y \geq 1 - \delta$  as censored observations. By default, no censoring is applied ( $\delta = 0$ ).

## Link-function

The linear predictor  $\eta$  is linked to the mean  $\mu$  using a default logit-link

$$\mu = \frac{\exp(\eta)}{1 + \exp(\eta)}.$$

## Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = s_i \exp(\theta)$$

where  $s = (s_i) > 0$  is a fixed scaling, and the prior is defined on  $\theta$ .

## Specification

- family = **beta**
- Required argument:  $y$
- Optional argument:  $s$  (argument **scale**, default all 1,  $s > 0$ )
- Optional argument: truncation limit  $0 \leq \delta < 1/2$  (argument **beta.truncation**,  $\delta = 0$  means no truncation).

## Hyperparameter specification and default values

**doc** The Beta likelihood

**hyper**

**theta**

**hyperid** 61001  
**name** precision parameter  
**short.name** phi  
**initial** 2.30258509299405  
**fixed** FALSE  
**prior** loggamma  
**param** 1 0.1  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**survival** FALSE

**discrete** FALSE

**link** default logit loga cauchit probit cloglog ccloglog loglog

**pdf** beta

## Example

In the following example we estimate the parameters in a simulated example.

```
n = 1000
w = runif(n, min = 0.25, max = 0.75)
phi = 5 * w
z = rnorm(n, sd=0.2)
eta = 1 + z
mu = exp(eta)/(1+exp(eta))
a = mu * phi
b = -mu * phi + phi
y = rbeta(n, a, b)

formula = y ~ 1 + z
r = inla(formula, data = data.frame(y, z, w),
        family = "beta", scale = w)
summary(r)
```

In this example we add truncation.

```
## the precision parameter in the beta distribution
phi = 5

## generate simulated data
n = 1000
z = rnorm(n, sd=.2)
eta = 1 + z
```

```

mu = exp(eta)/(1+exp(eta))
a = mu * phi
b = -mu * phi + phi
y = rbeta(n, a, b)

## this is the censoring
cens <- 0.05
y[y <= cens] <- 0
y[y >= 1-cens] <- 1

## estimate the model
formula = y ~ 1 + z
r = inla(formula, data = data.frame(y, z), family = "beta",
         control.family = list(beta.censor.value = cens))
summary(r)

```

## Notes

None.