Gaussian

Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} s\tau\,(y-\mu)^2\right)$$

for continuously responses y where

μ**:** is the the mean

τ**:** is the precision

s**:** is a fixed scaling, $s > 0$.

Link-function

The mean and variance of y are given as

$$\mu \quad \text{and} \qquad \sigma^2 = \frac{1}{s\tau}$$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The default behaviour is to represent the precision $\tau = \kappa_1$ where

$$\theta_1 = \log \kappa_1$$

and the prior is defined on θ_1.

The more general formulation have a second (fixed) hyperparameter θ_2 which determines a fixed offset $1/\kappa_2$, $\theta_2 = \log \kappa_2$, for the variance (scaling not included) of the response. In this case,

$$1/\tau = 1/\kappa_1 + 1/\kappa_2$$

or

$$\tau = \frac{1}{1/\kappa_1 + 1/\kappa_2}$$

In the case where $1/\kappa_2$ is zero, then $\tau = \kappa_1$ and we are back to the default behaviour. We use the convension that $1/\kappa_2$ is zero if $1/\kappa_2 <$ `.Machine$double.eps`,which is $\theta_2 \geq 36.05$ for common machines.

Specification

- `family="gaussian"`
- Required arguments: y and s (argument `scale`)

The scalings have default value 1.

There is also `family="stdgaussian"` for the case where the precision τ is fixed to 1 with no offset (but `scale` is still in place).

Hyperparameter spesification and default values

family="gaussian"

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hyper

- **theta1**
  - **hyperid** 65001
  - **name** log precision
  - **short.name** prec
  - **output.name** Precision for the Gaussian observations
  - **output.name.intern** Log precision for the Gaussian observations
  - **initial** 4
  - **fixed** FALSE
  - **prior** loggamma
  - **param** 1 5e-05
  - **to.theta** function(x) log(x)
  - **from.theta** function(x) exp(x)
- **theta2**
  - **hyperid** 65002
  - **name** log precision offset
  - **short.name** precoffset
  - **output.name** NOT IN USE
  - **output.name.intern** NOT IN USE
  - **initial** 72.0873067782343
  - **fixed** TRUE
  - **prior** none
  - **param**
  - **to.theta** function(x) log(x)
  - **from.theta** function(x) exp(x)

survival FALSE

discrete FALSE

link default identity logit loga cauchit log logoffset

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family="stdgaussian"

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hyper

survival FALSE

discrete FALSE

link default identity logit loga cauchit log logoffset

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Example

The first example estimate the parameters in a simulated example with Gaussian responses, giving τ a Gamma-prior with parameters $(1, 0.01)$ and initial value (for the optimisations) of $\exp(2.0)$. The second example shows the use of an fixed offset in the variance.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))

data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
        control.family = list(hyper = list(
                                    prec = list(
                                            prior = "loggamma",
                                            param = c(1.0,0.01),
                                            initial = 2))),
              scale=scale, keep=TRUE)
summary(result)

## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
         data = data.frame(y, x),
         control.family = list(
             hyper = list(precoffset = list(initial = log(1/var0)))),
         verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "l", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")
```

Notes

An error is given if θ_2 is not fixed.