

## Gaussian

### Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s\tau(y-\mu)^2\right)$$

for continuously responses  $y$  where

$\mu$ : is the the mean

$\tau$ : is the precision

$s$ : is a fixed scaling,  $s > 0$ .

### Link-function

The mean and variance of  $y$  are given as

$$\mu \quad \text{and} \quad \sigma^2 = \frac{1}{s\tau}$$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

### Hyperparameters

The default behaviour is to represent the precision  $\tau = \kappa_1$  where

$$\theta_1 = \log \kappa_1$$

and the prior is defined on  $\theta_1$ .

The more general formulation have a second (fixed) hyperparameter  $\theta_2$  which determines a fixed offset  $1/\kappa_2$ ,  $\theta_2 = \log \kappa_2$ , for the variance (scaling not included) of the response. In this case,

$$1/\tau = 1/\kappa_1 + 1/\kappa_2$$

or

$$\tau = \frac{1}{1/\kappa_1 + 1/\kappa_2}$$

In the case where  $1/\kappa_2$  is zero, then  $\tau = \kappa_1$  and we are back to the default behaviour. We use the convension that  $1/\kappa_2$  is zero if  $1/\kappa_2 < \text{Machine}\$double.eps$ , which is  $\theta_2 \geq 36.05$  for common machines.

### Specification

- family = **gaussian**
- Required arguments:  $y$  and  $s$  (argument **scale**)

The scalings have default value 1.

## Hyperparameter specification and default values

**doc** The Gaussian likelihood

**hyper**

**theta1**

**hyperid** 65001  
**name** log precision  
**short.name** prec  
**initial** 4  
**fixed** FALSE  
**prior** loggamma  
**param** 1 5e-05  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**theta2**

**hyperid** 65002  
**name** log precision offset  
**short.name** preoffset  
**initial** 72.0873067782343  
**fixed** TRUE  
**prior** none  
**param**  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**survival** FALSE

**discrete** FALSE

**link** default identity logit loga cauchit log logoffset

**pdf** gaussian

## Example

The first example estimate the parameters in a simulated example with Gaussian responses, giving  $\tau$  a Gamma-prior with parameters (1, 0.01) and initial value (for the optimisations) of  $\exp(2.0)$ . The second example shows the use of an fixed offset in the variance.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))
```

```

data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
              control.family = list(hyper = list(
                                    prec = list(
                                      prior = "loggamma",
                                      param = c(1.0,0.01),
                                      initial = 2))),
                                scale=scale, keep=TRUE)
summary(result)

## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
          data = data.frame(y, x),
          control.family = list(
            hyper = list(precoffset = list(initial = log(1/var0)))),
          verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "l", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")

```

## Notes

An error is given if  $\theta_2$  is not fixed.