

# Generalized Pareto distribution

## Parametrisation

The generalized Pareto (GP) distribution with positive shape parameter has cumulative distribution function

$$F(y; \sigma, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}, \quad y > 0,$$

for a continuous response  $y$  where

$\xi$ : is the tail parameter,  $\xi > 0$

$\sigma$ : is the scale parameter,  $\sigma > 0$

The limit for  $\xi \downarrow 0$  is  $F(y; \sigma, 0) = 1 - \exp(-y/\sigma)$ .

## Link function

The linear predictor  $\eta$  controls the  $\alpha$  quantile of the GP

$$P(y \leq q_\alpha) = \alpha$$

and  $q_\alpha = \exp(\eta)$ . The scaling  $\sigma$ , is then a function of  $(q_\alpha, \xi)$ , as

$$\sigma = \frac{\xi \exp(\eta)}{(1 - \alpha)^{-\xi} - 1}$$

## Hyperparameters

The GP model has one hyperparameter. The tail  $\xi > 0$  is represented as

$$\xi = \xi_{\text{low}} + (\xi_{\text{high}} - \xi_{\text{low}}) \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior is defined on  $\theta$ , with constant low and high values. The prior is FIXED to `pc.gevtail`, see `inla.doc("pc.gevtail")` for more info.

## Specification

- `family=gp`
- Required arguments:  $y$  and the quantile  $\alpha$ .

The quantile is given as `control.family=list(control.link=list(quantile= $\alpha$ ))`.

## Hyperparameter specification and default values

**doc** Generalized Pareto likelihood

**hyper**

**theta**

**hyperid** 101201

**name** tail

**short.name** xi

**initial** -4

**fixed** FALSE

**prior** pc.gevtail

**param** 7 0 0.5

**to.theta** function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) log(-(interval

**from.theta** function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) interval[1]

**status** experimental

**survival** FALSE

**discrete** TRUE

**link** default quantile

**pdf** genPareto

## Example

```
rgp = function(n, sigma, eta, alpha, xi = 0.001)
{
  if (missing(sigma)) {
    stopifnot(!missing(eta) && !missing(alpha))
    sigma = exp(eta) * xi / ((1.0 - alpha)^(-xi) - 1.0)
  }
  return (sigma / xi * (runif(n)^(-xi) - 1.0))
}
```

```
n = 300
x = runif(n)-0.5
eta = 1+x
alpha = 0.95
xi = 0.3
y = rgp(n, eta = eta, alpha = alpha, xi=xi)

r = inla(y ~ 1+x,
```

```

data = data.frame(y, x),
family = "gp",
control.family = list(control.link = list(quantile = alpha)),
control.predictor = list(compute=TRUE),
verbose=TRUE)

rx = range(c(r$summary.fitted.values$mean, exp(eta)))
plot(r$summary.fitted.values$mean, exp(eta),
      xlim = rx, ylim = rx)
abline(a=0,b=1)

```

## Notes

The prior for  $\xi$  is fixed to `pc.gevtail` or one that respect an given interval similarly. Note that the default prior (and the internal representation of it and its name) changed 2019/10/12.