The Kumaraswamy distribution

Parametrisation

The Kumaraswamy distribution is

$$f(y) = \alpha\beta y^{\alpha-1}(1-y^{\alpha})^{\beta-1}$$

for $0 < y < 1$ and $\alpha, \beta > 0$. The cummulative distribution function is

$$F(y) = 1 - (1-y^{\alpha})^{\beta}.$$

The parametrisation is given in terms of the quantile function

$$\kappa(q) = \left(1 - (1-q)^{1/\beta}\right)^{1/\alpha}$$

and the precision parameter ϕ,

$$\phi(q) = -\ln\left(1 - (1-q)^{1/\beta}\right)$$

for *fixed* value of $0 < q < 1$.

Link-function

The quantile κ to the linear predictor by

$$\text{logit}(\kappa) = \eta$$

using the default logit link-function.

Hyperparameters

The hyperparameter is

$$\phi = \exp(S\theta)$$

and the prior is defined on θ. The constant S currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of α can make a huge difference.

Specification

- family = `qkumar`
- Required arguments: y and the quantile q.

The quantile is given as `control.family=list(control.link = list(quantile=`q`))`.

Hyperparameter spesification and default values

doc A quantile version of the Kumar likelihood

hyper

theta

hyperid 60001
name precision parameter
short.name prec
initial 1

fixed FALSE

prior loggamma

param 1 0.1

to.theta `function(x, sc = 0.1) log(x) / sc`

from.theta `function(x, sc = 0.1) exp(sc * x)`

survival FALSE

discrete FALSE

link default logit loga cauchit

pdf qkumar

Example

```
rkumar = function(n, eta, phi, q=0.5)
{
    kappa = eta
    beta = log(1-q)/log(1-exp(-phi))
    alpha = log(1- (1-q)^(1/beta)) / log(kappa)
    u = runif(n)
    y = (1-u^(1/beta))^(1/alpha)
    return (y)
}

n = 100
q = 0.5
phi = 1
x = rnorm(n, sd = 1)
eta = inla.link.invlogit(1 + x)
y = rkumar(n, eta, phi, q)
r = inla(y ~ 1 + x,
    data = data.frame(y, x),
    family = "qkumar",
    control.family = list(control.link=list(quantile = q)))
summary(r)
```

Notes

None.