

## Zero-inflated models: Beta-Binomial

### Parameterisation

There is support for a further zero-inflated model of type 2 (see zero-inflated.pdf), the zero-inflated beta-binomial. It is only defined for type 2.

### Type 2

The likelihood is defined as

$$\text{Prob}(y \mid \dots) = p \times 1_{[y=0]} + (1 - p) \times \text{Beta-binomial}(y)$$

where:

$$p = 1 - \left( \frac{\exp(x)}{1 + \exp(x)} \right)^\alpha$$

### Link-function

As for the Binomial (see Zero-inflated.pdf).

### Hyperparameters

The Beta-binomial distribution has two arguments ( $\beta_1$  &  $\beta_2$ ) which we assume are a (specific) function of an underlying hyperparameter ( $\delta$ ) &  $x$ . There is a further hyperparameter,  $\alpha$ , governing zero-inflation where:

The parameter controlling the degree of overdispersion,  $\delta$ , is represented as

$$\theta_1 = \log(\delta)$$

and the prior is defined on  $\theta_1$ .

The zero-inflation parameter  $\alpha$ , is represented as

$$\theta_2 = \log(\alpha)$$

and the prior and initial value is is given for  $\theta_2$ .

### Specification

- family="zeroinflatedbetabinomial2"
- Required arguments: As for the zero-inflated-nbinomial2 likelihood.

### Hyperparameter spesification and default values

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hyper

theta1

hyperid 94001

name log alpha

short.name a

```

    output.name zero-probability parameter for zero-inflated betabinomial_2
    output.name.intern intern zero-probability parameter for zero-inflated betabinomial_2
    initial 0.693147180559945
    fixed FALSE
    prior gaussian
    param 0.693147180559945 1
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 94002
    name beta
    short.name b
    output.name overdispersion parameter for zero-inflated betabinomial_2
    output.name.intern intern overdispersion parameter for zero-inflated betabinomial_2
    initial 0
    fixed FALSE
    prior gaussian
    param 0 1
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

pdf zeroinflated

```

## Example

In the following we estimate the parameters in a simulated example.

Example-zero-inflated-beta-binomial2.R

```

nx = 1000                # number of x's to consider
n.trial = 20             # size of each binomial trial
x = rnorm(nx)            # generating x

delta = 10               #hyperparameter 1
p = exp(1+x)/(1+exp(1+x)) #hyperparameter 2
alpha = 2                #ZI parameter
q = p^alpha              #prob presence

beta_1=delta*p            #beta-bin parameter 1
beta_2=delta*(1-p)        #beta-bin parameter 2
rb = rbeta(nx, beta_1, beta_2, ncp = 0)

y = rep(0,nx)            #generating data

```

```

abs.pres = rbinom(nx,1,q)
y[abs.pres==1] = rbinom( sum(abs.pres>0), n.trial, rb[abs.pres==1])

formula = y ~ x +1
r = inla(formula, data = data.frame(x,y), family = "zeroinflatedbetabinomial2",
        control.family = list(hyper=list(a = list(prior = "flat", param=numeric(0)),
                                                b = list(prior = "flat", param=numeric(0)))),
        Ntrials = rep(n.trial, nx),
        verbose=TRUE)

```