

# Intercept-slope model

## Parametrization

The intercept-slope model is a convenient re-implementation of a commonly used construct, where

$$(a, b)$$

is bi-variate Gaussian with a Wishart prior for the precision matrix<sup>1</sup>, and various forms of

$$\gamma(a + bz), \tag{1}$$

where  $z$  is a covariate and  $\gamma$  is a (random) scaling, goes into the linear predictor. Replicates of  $(a, b)$  is indexed by *subject*,  $i = 1, \dots, n$ , and the various scaling of Eq. 1 by *strata*  $j = 1, \dots, m$ , leading to a model for (a subset of)

$$\{\gamma_j(a_i + b_i z_{ij}), \quad i = 1, \dots, n, \quad j = 1, \dots, m\},$$

as not all combinations need to be present.

## Hyperparameters

The hyperparameters are  $(\theta_1, \theta_2, \theta_3)$  as in the model “iid2d” (related to the precisions of  $a$  and  $b$ , and their correlation), and  $\theta_4 = \gamma_1, \dots, \theta_{53} = \gamma_{50}$ . Since  $m$  is defined in the input, only  $\gamma_1, \dots, \gamma_m$  are used.  $m$  is limited to  $m \leq 50$ . **Please note** that all  $\gamma_i$ ’s are by default **fixed** to 1.

## Specification

The is specified as

```
f(idx, model="intslope", hyper = ...,
  precision = exp(14),
  args.intslope = list(subject=i, strata=j, covariate = z))
```

The definition of the model is through the `args.intslope` argument, where `i` and `j` are factors/integers and `z` is numerical, all with same length  $N$ , say. The argument `idx`, index which row that is used for the linear predictor, hence values of `idx` must take integer values in the interval 1 to  $N$ . The precision argument, defines the tiny small noise added to each  $\gamma(a + bz)$  to avoid a singular joint model. The `subject` and `strata` argument, is converted internally into integers 1, 2, ..., using

```
subject = as.numerical(as.factor(subject))
strata   = as.numerical(as.factor(strata))
```

and the results is shown after this conversion.

## Hyperparameter specification and default values

**doc** Intecept-slope model with Wishart-prior

**hyper**

```
  theta1
    hyperid 16101
    name log precision1
```

---

<sup>1</sup>The documentation for the model “iid2d” gives the details of the definition of the parameterization of the precision matrix and the Wishart-prior.

```

    short.name prec1
    initial 4
    fixed FALSE
    prior wishart2d
    param 4 1 1 0
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 16102
    name log precision2
    short.name prec2
    initial 4
    fixed FALSE
    prior none
    param
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta3
    hyperid 16103
    name logit correlation
    short.name cor
    initial 4
    fixed FALSE
    prior none
    param
    to.theta function(x) log((1 + x) / (1 - x))
    from.theta function(x) 2 * exp(x) / (1 + exp(x)) - 1
theta4
    hyperid 16104
    name gamma1
    short.name g1
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 16105
    name gamma2
    short.name g2
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 16106

```

```

    name gamma3
    short.name g3
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 16107
    name gamma4
    short.name g4
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 16108
    name gamma5
    short.name g5
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 16109
    name gamma6
    short.name g6
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 16110
    name gamma7
    short.name g7
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta11

```

```

    hyperid 16111
    name gamma8
    short.name g8
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta12
    hyperid 16112
    name gamma9
    short.name g9
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta13
    hyperid 16113
    name gamma10
    short.name g10
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta14
    hyperid 16114
    name gamma11
    short.name g11
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta15
    hyperid 16115
    name gamma12
    short.name g12
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x

```

theta16

hyperid 16116  
name gamma13  
short.name g13  
initial 1  
fixed TRUE  
prior normal  
param 1 36  
to.theta function(x) x  
from.theta function(x) x

theta17

hyperid 16117  
name gamma14  
short.name g14  
initial 1  
fixed TRUE  
prior normal  
param 1 36  
to.theta function(x) x  
from.theta function(x) x

theta18

hyperid 16118  
name gamma15  
short.name g15  
initial 1  
fixed TRUE  
prior normal  
param 1 36  
to.theta function(x) x  
from.theta function(x) x

theta19

hyperid 16119  
name gamma16  
short.name g16  
initial 1  
fixed TRUE  
prior normal  
param 1 36  
to.theta function(x) x  
from.theta function(x) x

theta20

hyperid 16120  
name gamma17  
short.name g17  
initial 1  
fixed TRUE  
prior normal  
param 1 36  
to.theta function(x) x

```

    from.theta function(x) x
theta21
  hyperid 16121
  name gamma18
  short.name g18
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta22
  hyperid 16122
  name gamma19
  short.name g19
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta23
  hyperid 16123
  name gamma20
  short.name g20
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta24
  hyperid 16124
  name gamma21
  short.name g21
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta25
  hyperid 16125
  name gamma22
  short.name g22
  initial 1
  fixed TRUE
  prior normal
  param 1 36

```

```

    to.theta function(x) x
    from.theta function(x) x
theta26
  hyperid 16126
  name gamma23
  short.name g23
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta27
  hyperid 16127
  name gamma24
  short.name g24
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta28
  hyperid 16128
  name gamma25
  short.name g25
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta29
  hyperid 16129
  name gamma26
  short.name g26
  initial 1
  fixed TRUE
  prior normal
  param 1 36
  to.theta function(x) x
  from.theta function(x) x
theta30
  hyperid 16130
  name gamma27
  short.name g27
  initial 1
  fixed TRUE
  prior normal

```

```

    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 16131
    name gamma28
    short.name g28
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 16132
    name gamma29
    short.name g29
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 16133
    name gamma30
    short.name g30
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta34
    hyperid 16134
    name gamma31
    short.name g31
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta35
    hyperid 16135
    name gamma32
    short.name g32
    initial 1
    fixed TRUE

```



```

    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta36
    hyperid 16136
    name gamma33
    short.name g33
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta37
    hyperid 16137
    name gamma34
    short.name g34
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta38
    hyperid 16138
    name gamma35
    short.name g35
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta39
    hyperid 16139
    name gamma36
    short.name g36
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta40
    hyperid 16140
    name gamma37
    short.name g37
    initial 1

```

```

    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta41
    hyperid 16141
    name gamma38
    short.name g38
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 16142
    name gamma39
    short.name g39
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta43
    hyperid 16143
    name gamma40
    short.name g40
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 16144
    name gamma41
    short.name g41
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 16145
    name gamma42
    short.name g42

```

```

    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta46
    hyperid 16146
    name gamma43
    short.name g43
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta47
    hyperid 16147
    name gamma44
    short.name g44
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta48
    hyperid 16148
    name gamma45
    short.name g45
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta49
    hyperid 16149
    name gamma46
    short.name g46
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta50
    hyperid 16150
    name gamma47

```

```

    short.name g47
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 16151
    name gamma48
    short.name g48
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta52
    hyperid 16152
    name gamma49
    short.name g49
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta53
    hyperid 16153
    name gamma50
    short.name g50
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
constr FALSE
nrow.ncol FALSE
augmented FALSE
aug.factor 1
aug.constr
n.div.by
n.required FALSE
set.default.values TRUE
pdf intslope

```

## Example

```
library(mvtnorm)
n = 300
idx = 1:n
nstrata = 3
strata = sample(1:nstrata, n, replace=TRUE)
nsubject = n %% nstrata
subject = sample(1:nsubject, n, replace=TRUE)
z = rnorm(n)
gam = c(1, 1 + rnorm(nstrata-1, sd = 0.2))

rho = sqrt(3)/2
Sigma = matrix(c(1/1, NA, NA, 1/2), 2, 2)
Sigma[1,2] = Sigma[2,1] = rho*sqrt(Sigma[1,1]*Sigma[2,2])

ab = rmvnorm(nsubject, sigma=Sigma)
a = ab[,1]
b = ab[,2]
s = 0.01
y = gam[strata] * (a[subject] + z * b[subject]) + rnorm(n, s = 0.01)
r = inla(y ~ -1 + f(idx, model = "intslope",
  args.intslope = list(subject = subject,
    strata = strata,
    covariates = z),
  ## this is for nstrata = 3
  hyper = list(gamma1 = list(fixed = TRUE),
    gamma2 = list(fixed = FALSE),
    gamma3 = list(fixed = FALSE))),
  data = list(y = y,
    idx = idx,
    subject = subject,
    strata = strata,
    z = z),
  control.family = list(hyper = list(
    prec = list(initial = log(1/s^2),
      fixed=TRUE))))))

summary(r)
```

## Notes

- With  $n_s = \max(\text{subject})$ , the internal storage of this model is

$$(\gamma_{j_1}(a_{i_1} + z_1 b_{i_1}), \dots, \gamma_{j_N}(a_{i_N} + z_N b_{i_N}), a_1, \dots, a_{n_s}, b_1, \dots, b_{n_s}),$$

i.e. a vector of length  $N + 2n_s$ .