

# The Gamma-distribution

## Parametrisation

The Gamma-distribution has the following density

$$\pi(y) = \frac{b^a}{\Gamma(a)} y^{a-1} \exp(-by), \quad a > 0, \quad b > 0, \quad y > 0,$$

where  $E(y) = \mu = a/b$  and  $\text{Var}(y) = 1/\tau = a/b^2$ , where  $\tau$  is the precision and  $\mu$  is the mean. We will use the following parameterisation for the precision

$$\tau = (s\phi)/\mu^2$$

where  $\phi$  is the precision parameter (or  $1/\phi$  is the dispersion parameter) and  $s > 0$  is a fixed scaling (for the regression model), which gives this density

$$\pi(y) = \frac{1}{\Gamma(s\phi)} \left( \frac{(s\phi)}{\mu} \right)^{(s\phi)} y^{(s\phi)-1} \exp \left( -(s\phi) \frac{y}{\mu} \right)$$

## Link-function

The linear predictor  $\eta$  is linked to the mean  $\mu$  using a default log-link

$$\mu = \exp(\eta)$$

## Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = \exp(\theta)$$

and the prior is defined on  $\theta$ .

## Specification

- `family="gamma"` for regression models and `family="gamma.surv"` for survival models.
- Required arguments: for `gamma.surv`,  $y$  (to be given in a format by using `inla.surv()`), and for `gamma`,  $y$  and  $s$  (default value 1).

The scalings  $s$  is **not** used for `family="gamma.surv"`.

## Hyperparameter spesification and default values

**doc** The Gamma likelihood

**hyper**

**theta**

**hyperid** 58001

**name** precision parameter

**short.name** prec

**initial** 4.60517018598809

**fixed** FALSE

**prior** loggamma  
**param** 1 0.01  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**survival** FALSE

**discrete** FALSE

**link** default log quantile

**pdf** gamma

**doc** The Gamma likelihood (survival)

**hyper**

**theta1**

**hyperid** 58101  
**name** precision parameter  
**short.name** prec  
**initial** 0  
**fixed** FALSE  
**prior** loggamma  
**param** 1 0.01  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**theta2**

**hyperid** 58102  
**name** beta1  
**short.name** beta1  
**initial** -7  
**fixed** FALSE  
**prior** normal  
**param** -4 100  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta3**

**hyperid** 58103  
**name** beta2  
**short.name** beta2  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 100  
**to.theta** function(x) x  
**from.theta** function(x) x

**theta4**

```

hyperid 58104
name beta3
short.name beta3
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta5
hyperid 58105
name beta4
short.name beta4
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta6
hyperid 58106
name beta5
short.name beta5
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta7
hyperid 58107
name beta6
short.name beta6
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta8
hyperid 58108
name beta7
short.name beta7
initial 0

```

```

    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 58109
    name beta8
    short.name beta8
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 58110
    name beta9
    short.name beta9
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 58111
    name beta10
    short.name beta10
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

survival TRUE

discrete FALSE

status experimental

link default log neglog quantile

pdf gammasurv

```

## Example

In the following example we estimate the parameters in a simulated example.

```
n = 1000
x = rnorm(n)
eta = 1 + x
mu = exp(eta)
prec.scale = runif(n, min = 0.5, max = 2)
prec.par = 1.2
a = prec.par * prec.scale
b = mu / (prec.par * prec.scale)
y = rgamma(n, shape = a, scale = b)
r = inla(y ~ 1 + x, data = data.frame(y, x),
        scale = prec.scale, family = "gamma")
```

## Notes

None.