Proportional odds model

Parametrisation

The proportional odds model, is for discrete observations

$$y \in \{1, 2, \ldots, K\}, \qquad K > 1,$$

defined via the cummulative distribution function

$$F(k) = \text{Prob}(y \le k) = \frac{\exp(\gamma_k)}{1 + \exp(\gamma_k)}$$

where

$$\gamma_k = \alpha_k - \eta.$$

$\{\alpha_k\}$ is here increasing sequence of $K - 1$ cut-off points,

$$\alpha_0 = -\infty < \alpha_1 < \alpha_2 < \ldots < \alpha_{K-1} < \alpha_K = 1,$$

and η is the linear predictor. The likelihood for an observation is then

$$\text{Prob}(y = k) = F(k) - F(k-1).$$

Link-function

Not available.

Hyperparameters

The hyperparameters are $\theta_1, \ldots, \theta_{K-1}$, where

$$\alpha_1 = \theta_1,$$

and

$$\alpha_k = \alpha_{k-1} + \exp(\theta_k) = \theta_1 + \sum_{j=2}^{k} \exp(\theta_j)$$

for $k = 2, \ldots, K-1$. The posteriors for $\{\alpha_k\}$ must be found through simulations as shown in the example below.

Specification

- family = `pom`
- Required arguments: y (observations)

Number of classes, K is determined as the maximum of the observations. Empty classes are not allowed.

Example

In the following example we estimate the parameters in a simulated example.

```
rpom = function(alpha, eta)
{
    ## alpha: the cutpoints. eta: the linear predictor
    F = function(x) 1.0/(1+exp(-x))

    ns = length(eta)
    y = numeric(ns)
    nc = length(alpha) + 1

    for(k in 1:ns) {
        p = diff(c(0.0, F(alpha - eta[k]), 1.0))
        y[k] = sample(1:nc, 1, prob = p)
    }
    return (y)
}

n = 300
nsim = 1E5
x = rnorm(n, sd = 0.3)

eta = x
alpha = c(-1, 0, 0.5)
y = rpom(alpha, eta)
prior.alpha = 3 ## parameter in the Dirichlet prior
r = inla(y ~ -1 + x, data = data.frame(y, x, idx = 1:n), family = "pom",
         control.family = list(hyper = list(theta1 = list(param = prior.alpha))))
summary(r)

## compute the posterior for the cutpoints
theta = inla.hyperpar.sample(nsim, r, intern=TRUE)
nms = paste(paste0("theta", 1:length(alpha)), "for POM")
sim.alpha = matrix(NA, dim(theta)[1], length(alpha))
for(k in 1:length(alpha)) {
    if (k == 1) {
        sim.alpha[, k] = theta[, nms[1]]
    } else {
        sim.alpha[, k] = sim.alpha[, k-1] + exp(theta[, nms[k]])
    }
}
colnames(sim.alpha) = paste0("alpha", 1:length(alpha))
m1 = colMeans(sim.alpha)
m2 = colMeans(sim.alpha^2)
print(cbind(truth = alpha, estimate = m1, stdev = sqrt(m2 - m1^2)))

for(k in 1:length(alpha)) {
    d = density(sim.alpha[, k])
    if (k == 1) {
        plot(d, xlim = 1.2*range(c(sim.alpha)),
             ylim = c(0, 1.5 * max(d$y)), type="l", lty=k, lwd=2)
    } else {
        lines(d, xlim = range(c(sim.alpha)), lty = k, lwd=2)
    }
    abline(v = alpha[k], lty=k, lwd=2)
}
```

Notes

The prior for $\{\theta_k\}$ are fixed to the Dirichlet distribution for

$$\big(F^{-1}(\alpha_1), \quad F^{-1}(\alpha_2) - F^{-1}(\alpha_1), \quad F^{-1}(\alpha_3) - F^{-1}(\alpha_2), \quad \ldots, \quad 1 - F^{-1}(\alpha_{K-1})\big)$$

with a common scale parameter; see `inla.doc("dirichlet", sec="prior")`