Fractional Gaussian Noise (FGN)

Parametrization

The (stationary) FGN (Gaussian) process has correlation function at lag k

$$\rho(k) = |k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}$$

where H is the Hurst parameter or self-similarity parameter, which we assume to be

$$1/2 \leq H < 1.$$

so the process has long range properties for $H > 1/2$. The locations of the the process is fixed to $1, 2, \ldots, n$, where n is the dimension of the finite representation of the FGN process.

Hyperparameters

The marginal precision, τ, of the process is represented as

$$\tau = \exp(\theta_1)$$

The Hurst parameter H is represented as

$$H = \frac{1}{2} + \frac{1}{2}\frac{\exp(\theta_2)}{1+\exp(\theta_2)}$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The FGN model is specified as

```
f(<whatever>, model="fgn", order=<order>, hyper = <hyper>)
```

The parmeter `order` gives the order of the Markov approximation. Currently, only `order=3` is implemented.

Hyperparameter spesification and default values for `model="fgn"`

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hyper

theta1

hyperid 13101
name log precision
short.name prec
prior pc.prec
param 3 0.01
initial 1
fixed FALSE
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

theta2

- **hyperid** 13102
- **name** logit H
- **short.name** H
- **prior** pcfgnh
- **param** 0.9 0.1
- **initial** 2
- **fixed** FALSE
- **to.theta** `function(x) log((2*x-1)/(2*(1-x)))`
- **from.theta** `function(x) 0.5 + 0.5*exp(x)/(1+exp(x))`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 5

aug.constr 1

n.div.by

n.required FALSE

set.default.values TRUE

order.default 4

order.defined 3 4

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Hyperparameter spesification and default values for `model="fgn2"`

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hyper

- **theta1**
  - **hyperid** 13101
  - **name** log precision
  - **short.name** prec
  - **prior** pc.prec
  - **param** 3 0.01
  - **initial** 1
  - **fixed** FALSE
  - **to.theta** `function(x) log(x)`
  - **from.theta** `function(x) exp(x)`
- **theta2**
  - **hyperid** 13102
  - **name** logit H
  - **short.name** H

prior pcfgnh
param 0.9 0.1
initial 2
fixed FALSE
to.theta `function(x) log((2*x-1)/(2*(1-x)))`
from.theta `function(x) 0.5 + 0.5*exp(x)/(1+exp(x))`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 4

aug.constr 1

n.div.by

n.required FALSE

set.default.values TRUE

order.default 4

order.defined 3 4

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Example

```
library(FGN)
n = 1000
H = 0.77
y = SimulateFGN(n, H)
y = y - mean(y)
r = inla(y ~ -1 + f(idx, model="fgn"),
         data = data.frame(y, idx=1:n),
         control.family =list(hyper = list(prec = list(initial = 12, fixed=TRUE))))
print(c(MLE=FitFGN(y, demean=TRUE)$H,
        Post.mean=r$summary.hyperpar[2,"mean"],
        Post.mode=r$summary.hyperpar[2,"mode"]))
```

Notes

In the example above, then the `f(idx,model="fgn")` object will expand into a Gaussian of length `(order + 1)*n`. The first n elements is the FGN model (which is of interrest), then there are `order` vector of AR1 processes each of length n, and the sum of these AR1 processes is used to represent the FGN.

Another alternative, is `f(idx,model="fgn2")` object will expand into a Gaussian of length `order*n`, which are the cummulative sums of the the `order` vector of AR1 processes each of length n. If `order==3`, with weighted AR1 processes (and with the given precision), x, xx and xxx, then `model="fgn2"` return the vector $(x + xx + xxx, xx + xxx, xxx)$ where $\phi_x < \phi_{xx} < \phi_{xxx}$.

The PC-prior for H take two arguments (U, α) where $\text{Prob}(U < H < 1) = \alpha$.