Skew-Normal likelihood

Parametrisation

The standardised Skew-Normal distribution is

$$f(z) = \frac{2}{\omega_\alpha}\phi\left(\frac{z-\xi_\alpha}{\omega_\alpha}\right)\Phi\left(\alpha\frac{z-\xi_\alpha}{\omega_\alpha}\right)$$

where ω_α and ξ_α are so that the mean is zero and variance is one, and they depends both on the skewness parameter α.

The skew-normal likelihood is defined as the density wrt y, where

$$z = (y-\eta)\sqrt{w\tau} \;\sim\; f(z)$$

and

η**:** is the the linear predictor

τ**:** is the precision

w**:** is a fixed scale or weight, $w > 0$,

Link-function

The mean equals the linear predictor

$$\mu = \eta$$

Hyperparameters

The precision is represented as

$$\theta_1 = \log\tau$$

and the prior is defined on θ_1.

The (standardised) skewness γ, is represented as

$$\gamma = 0.988(2\frac{\exp(\theta_2)}{1+\exp(\theta_2)} - 1)$$

and the prior is defined on θ_2. The standardised skewness depends on α as

$$\gamma = \frac{4-\pi}{2}\frac{\left(\delta\sqrt{2/\pi}\right)^3}{(1-2\delta^2/\pi)^{3/2}}, \qquad \delta = \frac{\alpha}{\sqrt{1+\alpha^2}}$$

Specification

- family = `sn`
- Required arguments: y and w (keyword `scale`, and $w = 1$ by default).

Hyperparameter specification and default values

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hyper

theta1

hyperid 74001
name log precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

theta2

hyperid 74002
name logit skewness
short.name skewness
initial 0.00123456789
fixed FALSE
prior pc.sn
param 10
to.theta `function(x, skew.max = 0.988) log((1 + x / skew.max) / (1 - x / skew.max))`
from.theta `function(x, skew.max = 0.988) skew.max * (2 * exp(x) / (1 + exp(x)) - 1)`

status experimental

survival FALSE

discrete FALSE

link default identity

pdf sn

Example

```
library(sn)
set.seed(246)
n = 300
x = rnorm(n, sd = 1)
eta = 1+x
skewness = 0.25
y = numeric(n)
prec <- 100
for(i in 1:n) {
    ## map moments to sn-parameters c(xi, omega, alpha)
    param = INLA:::inla.sn.reparam(moments = c(eta[i], 1/prec, skewness))
```

```
    y[i] = rsn(1, xi=param$xi, omega = param$omega, alpha = param$alpha)
}

r = inla(y ~ 1+x,
         family = "sn",
         data = data.frame(y, x),
         control.family = list(
             hyper = list(prec = list(
                                 prior = "pc.prec",
                                 param = c(3, 0.01)))))
summary(r)
```

Notes

- This implementation replaces older ones ("sn" and "sn2") from 16^{th} September 2020.
- A $N(a, 0)$ prior is interpreted as a constant prior with density equal to one.