

# Weibull

## Parametrisation

The Weibull distribution is (`variant=0`)

$$f(y) = \alpha y^{\alpha-1} \lambda \exp(-\lambda y^\alpha), \quad \alpha > 0, \quad \lambda > 0$$

and (`variant=1`)

$$f(y) = \alpha y^{\alpha-1} \lambda^\alpha \exp(-(\lambda y)^\alpha), \quad \alpha > 0, \quad \lambda > 0$$

where

$\alpha$ : shape parameter.

## Link-function

The parameter  $\lambda$  is linked to the linear predictor as:

$$\lambda = \exp(\eta)$$

## Hyperparameters

The  $\alpha$  parameter is represented as

$$\alpha = \exp(S\theta)$$

and the prior is defined on  $\theta$ . The constant  $S$  currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of  $\alpha$  can make a huge difference.

## Specification

- `family="weibull"` for regression and `family="weibull.surv"` for survival
- Required arguments:  $y$  (to be given using `inla.surv()` for survival models), and `variant=0` (default) or 1 to define the parameterisation.

## Hyperparameter spesification and default values

### weibull

```
doc The Weibull likelihood
hyper
  theta
    hyperid 79001
    name log alpha
    short.name alpha
    output.name alpha parameter for weibull
    output.name.intern alpha_intern for weibull
    initial -2
    fixed FALSE
    prior pc.alphaw
    param 5
    to.theta function(x, sc = 0.1) log(x) / sc
    from.theta function(x, sc = 0.1) exp(sc * x)
```

```
survival FALSE
discrete FALSE
link default log neglog quantile
pdf weibull
```

## weibullsurv

doc The Weibull likelihood (survival)

hyper

theta

```
hyperid 79101
name log alpha
short.name alpha
output.name alpha parameter for weibullsurv
output.name.intern alpha_intern for weibullsurv
initial -2
fixed FALSE
prior pc.alphaw
param 5
to.theta function(x, sc = 0.1) log(x) / sc
from.theta function(x, sc = 0.1) exp(sc * x)
```

theta2

```
hyperid 79102
name beta1
short.name beta1
output.name beta1 for Weibull-Cure
output.name.intern beta1 for Weibull-Cure
initial -7
fixed FALSE
prior normal
param -4 100
to.theta function(x) x
from.theta function(x) x
```

theta3

```
hyperid 79103
name beta2
short.name beta2
output.name beta2 for Weibull-Cure
output.name.intern beta2 for Weibull-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
```

```

theta4
  hyperid 79104
  name beta3
  short.name beta3
  output.name beta3 for Weibull-Cure
  output.name.intern beta3 for Weibull-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta5
  hyperid 79105
  name beta4
  short.name beta4
  output.name beta4 for Weibull-Cure
  output.name.intern beta4 for Weibull-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta6
  hyperid 79106
  name beta5
  short.name beta5
  output.name beta5 for Weibull-Cure
  output.name.intern beta5 for Weibull-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta7
  hyperid 79107
  name beta6
  short.name beta6
  output.name beta6 for Weibull-Cure
  output.name.intern beta6 for Weibull-Cure
  initial 0
  fixed FALSE
  prior normal

```

```

    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 79108
    name beta7
    short.name beta7
    output.name beta7 for Weibull-Cure
    output.name.intern beta7 for Weibull-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 79109
    name beta8
    short.name beta8
    output.name beta8 for Weibull-Cure
    output.name.intern beta8 for Weibull-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 79110
    name beta9
    short.name beta9
    output.name beta9 for Weibull-Cure
    output.name.intern beta9 for Weibull-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 79111
    name beta10
    short.name beta10
    output.name beta10 for Weibull-Cure
    output.name.intern beta10 for Weibull-Cure

```

```

initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x

survival TRUE
discrete FALSE
link default log neglog quantile
pdf weibull

```

## Example

In the following example we estimate the parameters in a simulated case

```

n = 1000
alpha = 1.1
beta = 2.2
x = c(scale(runif(n)))
eta = 1+beta*x
lambda = exp(eta)

for(variant in 0:1) {
  y = rweibull(n,
              shape= alpha,
              scale= if (variant == 0)
                    lambda^(-1/alpha)
              else
                    1/lambda)

  print(paste("VARIANT=", variant))
  event = rep(1,n)
  data = list(y=y, event=event, x=x)

  formula=inla.surv(y,event)~ x
  r=inla(formula,
        family ="weibullsurv",
        data=data,
        control.family = list(list(variant = variant)))
  print("SURV")
  print(summary(r))

  formula= y ~ x
  r=inla(formula,
        family ="weibull",
        data=data,
        control.family = list(list(variant = variant)))
  print("REGRESSION")
  print(summary(r))
}

```

## Notes

- Weibullsurv model can be used for right censored, left censored, interval censored data. If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observations, `time = time / max(time)` or similar.