qLogLogistic likelihood

Parametrisation

The LogLogistic distribution has cumulative distribution function

$$F_0(y) = \frac{1}{1 + \lambda y^{-\alpha}}, \qquad y > 0$$

if `variant=0`, or

$$F_1(y) = \frac{1}{1 + (\lambda y)^{-\alpha}}, \qquad y > 0$$

if `variant=1`, where

$\alpha > 0$ is a shape parameter, and

$\lambda > 0$ is a scale parameter.

The λ is defined implicitely through the quantile, as

$$F_0(y_q) = q, \quad \text{or} \quad F_q(y_q) = q, \quad 0 < q < 1$$

and the linear predictor is defined on y_q.

Link-functions

The parameter λ is linked to the linear predictor, implicitely through

$$y_q = \exp(\eta)$$

Hyperparameters

The α parameter is represented as

$$\theta = \log \alpha$$

and the prior is defined on θ.

Specification

- `family` equals `qloglogistic` (regression) or `qloglogisticsurv` (survival)
- `variant=0` (default) or 1, chosing between parameterisation F_0 or F_1.
- Required arguments: y (regression) or an `inla.surv`-object using `inla.surv()` (for survival data), and quantile= q.

Hyperparameter spesification and default values

Regression:

doc A quantile loglogistic likelihood

hyper

theta

hyperid 60011
name log alpha

short.name alpha
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

survival FALSE

discrete FALSE

link default log neglog

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Survival:

doc A quantile loglogistic likelihood (survival)

hyper

theta

hyperid 60021
name log alpha
short.name alpha
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

survival TRUE

discrete FALSE

link default log neglog

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Example

In the following example we estimate the parameters in a simulated case

```
lam_loglogistic = function(yq, alpha, q, variant = 0)
{
    if (variant == 0) {
        lambda = yq^alpha * (1/q-1)
    } else if (variant == 1) {
        lambda = 1/yq * (1/(1/q-1))^(1/alpha)
    } else
        stop("ERR")
    return (lambda)
```

```
}


rloglogistic = function(n,  lambda,  alpha, variant=0)
{
    u = runif(n)
    if (variant == 0) {
        y = (lambda/(1.0/u - 1.0))^(1.0/alpha)
    } else if (variant == 1) {
        y = (1.0/(1.0/u -1.0))^(1.0/alpha) / lambda
    } else {
        stop("ERROR")
    }
}

n = 500
alpha = 2.1
x = c(scale(runif(n)))
eta = 1.1+2.2*x
yq = exp(eta)

for(variant in 0:1) {
    for(q in c(0.2, 0.8)) {

        print(paste("variant=", variant, "quantile=", q))
        lambda = lam_loglogistic(yq, alpha, q, variant=variant)
        y = rloglogistic(n,
                         lambda = lambda,
                         alpha = alpha,
                         variant = variant)

        formula = y ~ 1 + x
        rr=inla(formula,
               family ="qloglogistic",
               data=data.frame(y, x),
               control.family = list(list(variant = variant, control.link = list(quantile = q
        print("REGRESSION")
        print(summary(rr))

        event = rep(1,n)
        formula=inla.surv(y,event) ~ 1 + x
        r=inla(formula,
               family ="qloglogisticsurv",
               data = list(y=y, event=event, x=x),
               control.family = list(list(variant = variant, control.link = list(quantile = q
        print("SURVIVAL")
        print(summary(r))
    }
}
```

Notes

- Loglogisticsurv model can be used for right censored, left censored, interval censored data. If the observed times y are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observatios, `time = time / max(time)` or similar.