

# The Berkson Measurement Error (MEB) model

## Parametrization

This is an implementation of the Berkson measurement error model for a fixed effect. The observed covariate is  $w$  but it is  $x$  that goes into the linear predictor

$$\eta = \dots + \beta x + \dots ,$$

where  $x = w + u$ . The error term  $u$  is Gaussian with prior  $\mathcal{N}(0, \tau_u \mathbf{D})^1$ , where  $\tau_u$  is the observational precision of the error  $\text{Prec}(u|x)$  with possible heteroscedasticity, encoded in the entries  $d_i$  of the diagonal matrix  $\mathbf{D}$ . The vector  $s$  contains the fixed scalings  $s = (d_1, \dots, d_n)$  (with  $n$  the number of data points).

## Hyperparameters

This model has 2 hyperparameters,  $\theta = (\theta_1, \theta_2)$ . The hyperparameter specification is as follows:

$$\theta_1 = \beta$$

and the prior is defined on  $\theta_1$ ,

$$\theta_2 = \log(\tau_u)$$

and the prior is defined on  $\theta_2$ .

## Specification

The MEB is specified inside the `f()` function as

```
f(w, [<weights>], model="meb", scale = <s>, values= <w>, hyper = <hyper>)
```

Here, `w` are the observed covariates, and the fixed scaling of the observational precision is given in argument `scale`. If the argument `scale` is not given, then  $s$  is set to 1.

Note that only the unique values of `w` are used, so if two or more elements of `w` are *identical*, then they refer to the *same* element in the covariate  $x$ . If data points with identical  $w$  values belong to different  $x$  values (e.g., different individuals), please add a *tiny* random value to  $w$  to make this difference obvious to the model.

## Hyperparameter specification and default values

```
doc Berkson measurement error model
```

```
hyper
```

```
  theta1
```

```
    hyperid 3001
```

```
    name beta
```

```
    short.name b
```

```
    prior gaussian
```

```
    param 1 0.001
```

```
    initial 1
```

```
    fixed FALSE
```

```
    to.theta function(x) x
```

---

<sup>1</sup>Note: The second argument in  $\mathcal{N}(,)$  is the precision not the variance.

```

    from.theta function(x) x
  theta2
    hyperid 3002
    name prec.u
    short.name prec
    prior loggamma
    param 1 1e-04
    initial 6.90775527898214
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

  constr FALSE

  nrow.ncol FALSE

  augmented FALSE

  aug.factor 1

  aug.constr

  n.div.by

  n.required FALSE

  set.default.values FALSE

  pdf meb

```

## Example

```

n = 100
beta = 2
w = rnorm(n)
prec.u = 1
prec.y = 1
## heteroscedastic scaling
s = runif(n,min=0,max=1)
## true but unobserved covariate
x = w + rnorm(n, sd = 1/sqrt(s*prec.u))
y = 1 + beta*x + rnorm(n, sd = 1/sqrt(prec.y))

## prior parameters
prior.beta = c(0, 0.0001)
prior.prec.u = c(10, 9/prec.u)
prior.prec.y = c(10, 9/prec.y)

formula = y ~ f(w, model="meb", scale=s, values=w,
  hyper = list(
    beta = list(
      prior = "gaussian",

```

```

        param = prior.beta,
        fixed = FALSE
    ),
    prec.u = list(
        prior = "loggamma",
        param = prior.prec.u,
        initial = log(prec.u),
        fixed = FALSE
    )
)
)

r = inla(formula, data = data.frame(y, w, s),
        family = "gaussian",
        control.family = list(
            hyper = list(
                prec = list(param = prior.prec.y,
                            fixed = FALSE
                )
            )
        )
)

```

## Notes

- INLA provides the posteriors of  $\nu_i = \beta x_i$  and NOT  $x_i$ .
- The posteriors of  $\nu_i$  come (default) in the order given by the sorted (from low to high) values of **w**. The entry **\$ID** gives the mapping.
- The option **scale** defines the scaling in the same order as argument **values**. It is therefore advised to also give argument **values** when **scale** is used to be sure that they are consistent.