

## Correlated random effects: iidkd

This model is available for dimensions  $k = 2$ , to 20. We describe in detail the case for  $k = 3$  as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a different and more efficient parameterisation.

### Parametrization

The  $(k = 3)$ -dimensional Normal-Wishard model is used if one want to define three vectors of “random effects”,  $u$  and  $v$  and  $w$ , say, for which  $(u_i, v_i, w_i)$  are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

where the covariance matrix  $\mathbf{W}^{-1}$  is parameterised as  $\mathbf{W} = \mathbf{L}\mathbf{L}^T$ , where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \quad (1)$$

and  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$  can take any value. The number of hyperparameters are  $k(k+1)/2$ , which is 3, 6, 10, 15, 21, 28, 36, 45, 55, ..., for  $k = 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots$

For these models the precision matrix  $\mathbf{W}$  is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(k+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > k+1$$

and

$$c = 2^{(rk)/2} |\mathbf{R}|^{-r/2} \pi^{(k(k-1))/4} \prod_{j=1}^k \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (k+1)).$$

### Hyperparameters

The hyperparameters are  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ .

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters  $\{\theta_i\}$ , and the conversion into interpretable quantities can be done using simulation as described below.

The prior for  $\theta$  is **fixed** to be `wishartkd`, and number of prior parameters required are  $1 + k(k+1)/2$ . By default the prior-parameters are

$$(r = 100, \underbrace{1, \dots, 1}_{k \text{ times}}, 0, \dots, 0)$$

The model `iidkd` is specified as

where  $\text{order} = k = 3$ , and the `iidkd` model is represented internally as one vector of length  $n$ ,

where  $n = 3m$ , and  $n$  is the (required) argument in  $\mathbf{f}()$ .

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

(**Note:** The value “1048576” is just a code for “replace this by the default value”. As the default value depends on **order**, this was the easy way out for the moment.)

hyper

```
hyperid 29101  
name theta1  
short.name theta1  
initial 1048576  
fixed FALSE  
prior wishartkd  
param 21 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576 1048576  
to.theta function(x) x  
from.theta function(x) x
```

```
hyperid 29102
name theta2
short.name theta2
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
```

```
hyperid 29103
name theta3
short.name theta3
initial 1048576
fixed FALSE
```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 29104
    name theta4
    short.name theta4
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 29105
    name theta5
    short.name theta5
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 29106
    name theta6
    short.name theta6
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 29107
    name theta7
    short.name theta7
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

theta8

hyperid 29108  
name theta8  
short.name theta8  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

theta9

hyperid 29109  
name theta9  
short.name theta9  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

theta10

hyperid 29110  
name theta10  
short.name theta10  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

theta11

hyperid 29111  
name theta11  
short.name theta11  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

theta12

hyperid 29112  
name theta12  
short.name theta12

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta13
    hyperid 29113
    name theta13
    short.name theta13
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta14
    hyperid 29114
    name theta14
    short.name theta14
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta15
    hyperid 29115
    name theta15
    short.name theta15
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta16
    hyperid 29116
    name theta16
    short.name theta16
    initial 1048576
    fixed FALSE
    prior none
    param

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    to.theta function(x) x
    from.theta function(x) x
theta17
  hyperid 29117
  name theta17
  short.name theta17
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta18
  hyperid 29118
  name theta18
  short.name theta18
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta19
  hyperid 29119
  name theta19
  short.name theta19
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta20
  hyperid 29120
  name theta20
  short.name theta20
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta21
  hyperid 29121

```

```

    name theta21
    short.name theta21
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta22
    hyperid 29122
    name theta22
    short.name theta22
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta23
    hyperid 29123
    name theta23
    short.name theta23
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta24
    hyperid 29124
    name theta24
    short.name theta24
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta25
    hyperid 29125
    name theta25
    short.name theta25
    initial 1048576
    fixed FALSE

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    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta26
    hyperid 29126
    name theta26
    short.name theta26
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta27
    hyperid 29127
    name theta27
    short.name theta27
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta28
    hyperid 29128
    name theta28
    short.name theta28
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta29
    hyperid 29129
    name theta29
    short.name theta29
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

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#### theta30

hyperid 29130  
name theta30  
short.name theta30  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta31

hyperid 29131  
name theta31  
short.name theta31  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta32

hyperid 29132  
name theta32  
short.name theta32  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta33

hyperid 29133  
name theta33  
short.name theta33  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta34

hyperid 29134  
name theta34  
short.name theta34

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    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta35
    hyperid 29135
    name theta35
    short.name theta35
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta36
    hyperid 29136
    name theta36
    short.name theta36
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta37
    hyperid 29137
    name theta37
    short.name theta37
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta38
    hyperid 29138
    name theta38
    short.name theta38
    initial 1048576
    fixed FALSE
    prior none
    param

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    to.theta function(x) x
    from.theta function(x) x
theta39
  hyperid 29139
  name theta39
  short.name theta39
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta40
  hyperid 29140
  name theta40
  short.name theta40
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta41
  hyperid 29141
  name theta41
  short.name theta41
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta42
  hyperid 29142
  name theta42
  short.name theta42
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta43
  hyperid 29143

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```

    name theta43
    short.name theta43
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 29144
    name theta44
    short.name theta44
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 29145
    name theta45
    short.name theta45
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta46
    hyperid 29146
    name theta46
    short.name theta46
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta47
    hyperid 29147
    name theta47
    short.name theta47
    initial 1048576
    fixed FALSE

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    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta48
    hyperid 29148
    name theta48
    short.name theta48
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta49
    hyperid 29149
    name theta49
    short.name theta49
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta50
    hyperid 29150
    name theta50
    short.name theta50
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta51
    hyperid 29151
    name theta51
    short.name theta51
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

**theta52**

hyperid 29152  
name theta52  
short.name theta52  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta53**

hyperid 29153  
name theta53  
short.name theta53  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta54**

hyperid 29154  
name theta54  
short.name theta54  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta55**

hyperid 29155  
name theta55  
short.name theta55  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta56**

hyperid 29156  
name theta56  
short.name theta56

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta57
    hyperid 29157
    name theta57
    short.name theta57
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta58
    hyperid 29158
    name theta58
    short.name theta58
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta59
    hyperid 29159
    name theta59
    short.name theta59
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta60
    hyperid 29160
    name theta60
    short.name theta60
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta61
  hyperid 29161
  name theta61
  short.name theta61
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta62
  hyperid 29162
  name theta62
  short.name theta62
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta63
  hyperid 29163
  name theta63
  short.name theta63
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta64
  hyperid 29164
  name theta64
  short.name theta64
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta65
  hyperid 29165

```



```

    name theta65
    short.name theta65
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta66
    hyperid 29166
    name theta66
    short.name theta66
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta67
    hyperid 29167
    name theta67
    short.name theta67
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta68
    hyperid 29168
    name theta68
    short.name theta68
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta69
    hyperid 29169
    name theta69
    short.name theta69
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta70
    hyperid 29170
    name theta70
    short.name theta70
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta71
    hyperid 29171
    name theta71
    short.name theta71
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta72
    hyperid 29172
    name theta72
    short.name theta72
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta73
    hyperid 29173
    name theta73
    short.name theta73
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

**theta74**

hyperid 29174  
name theta74  
short.name theta74  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta75**

hyperid 29175  
name theta75  
short.name theta75  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta76**

hyperid 29176  
name theta76  
short.name theta76  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta77**

hyperid 29177  
name theta77  
short.name theta77  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta78**

hyperid 29178  
name theta78  
short.name theta78

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta79
    hyperid 29179
    name theta79
    short.name theta79
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta80
    hyperid 29180
    name theta80
    short.name theta80
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta81
    hyperid 29181
    name theta81
    short.name theta81
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta82
    hyperid 29182
    name theta82
    short.name theta82
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta83
  hyperid 29183
  name theta83
  short.name theta83
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta84
  hyperid 29184
  name theta84
  short.name theta84
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta85
  hyperid 29185
  name theta85
  short.name theta85
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta86
  hyperid 29186
  name theta86
  short.name theta86
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta87
  hyperid 29187

```

```

    name theta87
    short.name theta87
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta88
    hyperid 29188
    name theta88
    short.name theta88
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta89
    hyperid 29189
    name theta89
    short.name theta89
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta90
    hyperid 29190
    name theta90
    short.name theta90
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta91
    hyperid 29191
    name theta91
    short.name theta91
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta92
    hyperid 29192
    name theta92
    short.name theta92
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta93
    hyperid 29193
    name theta93
    short.name theta93
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta94
    hyperid 29194
    name theta94
    short.name theta94
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta95
    hyperid 29195
    name theta95
    short.name theta95
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

**theta96**

hyperid 29196  
name theta96  
short.name theta96  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta97**

hyperid 29197  
name theta97  
short.name theta97  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta98**

hyperid 29198  
name theta98  
short.name theta98  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta99**

hyperid 29199  
name theta99  
short.name theta99  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

**theta100**

hyperid 29200  
name theta100  
short.name theta100



```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta101
    hyperid 29201
    name theta101
    short.name theta101
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta102
    hyperid 29202
    name theta102
    short.name theta102
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta103
    hyperid 29203
    name theta103
    short.name theta103
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta104
    hyperid 29204
    name theta104
    short.name theta104
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta105
  hyperid 29205
  name theta105
  short.name theta105
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta106
  hyperid 29206
  name theta106
  short.name theta106
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta107
  hyperid 29207
  name theta107
  short.name theta107
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta108
  hyperid 29208
  name theta108
  short.name theta108
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta109
  hyperid 29209

```

```

    name theta109
    short.name theta109
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta110
    hyperid 29210
    name theta110
    short.name theta110
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta111
    hyperid 29211
    name theta111
    short.name theta111
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta112
    hyperid 29212
    name theta112
    short.name theta112
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta113
    hyperid 29213
    name theta113
    short.name theta113
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta114
    hyperid 29214
    name theta114
    short.name theta114
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta115
    hyperid 29215
    name theta115
    short.name theta115
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta116
    hyperid 29216
    name theta116
    short.name theta116
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta117
    hyperid 29217
    name theta117
    short.name theta117
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta118
  hyperid 29218
  name theta118
  short.name theta118
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta119
  hyperid 29219
  name theta119
  short.name theta119
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta120
  hyperid 29220
  name theta120
  short.name theta120
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta121
  hyperid 29221
  name theta121
  short.name theta121
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta122
  hyperid 29222
  name theta122
  short.name theta122

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta123
    hyperid 29223
    name theta123
    short.name theta123
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta124
    hyperid 29224
    name theta124
    short.name theta124
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta125
    hyperid 29225
    name theta125
    short.name theta125
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta126
    hyperid 29226
    name theta126
    short.name theta126
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta127
  hyperid 29227
  name theta127
  short.name theta127
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta128
  hyperid 29228
  name theta128
  short.name theta128
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta129
  hyperid 29229
  name theta129
  short.name theta129
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta130
  hyperid 29230
  name theta130
  short.name theta130
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta131
  hyperid 29231

```

```

    name theta131
    short.name theta131
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta132
    hyperid 29232
    name theta132
    short.name theta132
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta133
    hyperid 29233
    name theta133
    short.name theta133
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta134
    hyperid 29234
    name theta134
    short.name theta134
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta135
    hyperid 29235
    name theta135
    short.name theta135
    initial 1048576
    fixed FALSE

```



```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta136
    hyperid 29236
    name theta136
    short.name theta136
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta137
    hyperid 29237
    name theta137
    short.name theta137
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta138
    hyperid 29238
    name theta138
    short.name theta138
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta139
    hyperid 29239
    name theta139
    short.name theta139
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta140
  hyperid 29240
  name theta140
  short.name theta140
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta141
  hyperid 29241
  name theta141
  short.name theta141
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta142
  hyperid 29242
  name theta142
  short.name theta142
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta143
  hyperid 29243
  name theta143
  short.name theta143
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta144
  hyperid 29244
  name theta144
  short.name theta144

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta145
    hyperid 29245
    name theta145
    short.name theta145
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta146
    hyperid 29246
    name theta146
    short.name theta146
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta147
    hyperid 29247
    name theta147
    short.name theta147
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta148
    hyperid 29248
    name theta148
    short.name theta148
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta149
  hyperid 29249
  name theta149
  short.name theta149
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta150
  hyperid 29250
  name theta150
  short.name theta150
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta151
  hyperid 29251
  name theta151
  short.name theta151
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta152
  hyperid 29252
  name theta152
  short.name theta152
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta153
  hyperid 29253

```

```

name theta153
short.name theta153
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta154
hyperid 29254
name theta154
short.name theta154
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta155
hyperid 29255
name theta155
short.name theta155
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta156
hyperid 29256
name theta156
short.name theta156
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta157
hyperid 29257
name theta157
short.name theta157
initial 1048576
fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta158
    hyperid 29258
    name theta158
    short.name theta158
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta159
    hyperid 29259
    name theta159
    short.name theta159
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta160
    hyperid 29260
    name theta160
    short.name theta160
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta161
    hyperid 29261
    name theta161
    short.name theta161
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

#### theta162

hyperid 29262  
name theta162  
short.name theta162  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta163

hyperid 29263  
name theta163  
short.name theta163  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta164

hyperid 29264  
name theta164  
short.name theta164  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta165

hyperid 29265  
name theta165  
short.name theta165  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta166

hyperid 29266  
name theta166  
short.name theta166

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta167
    hyperid 29267
    name theta167
    short.name theta167
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta168
    hyperid 29268
    name theta168
    short.name theta168
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta169
    hyperid 29269
    name theta169
    short.name theta169
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta170
    hyperid 29270
    name theta170
    short.name theta170
    initial 1048576
    fixed FALSE
    prior none
    param

```



```

    to.theta function(x) x
    from.theta function(x) x
theta171
  hyperid 29271
  name theta171
  short.name theta171
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta172
  hyperid 29272
  name theta172
  short.name theta172
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta173
  hyperid 29273
  name theta173
  short.name theta173
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta174
  hyperid 29274
  name theta174
  short.name theta174
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta175
  hyperid 29275

```

```

    name theta175
    short.name theta175
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta176
    hyperid 29276
    name theta176
    short.name theta176
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta177
    hyperid 29277
    name theta177
    short.name theta177
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta178
    hyperid 29278
    name theta178
    short.name theta178
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta179
    hyperid 29279
    name theta179
    short.name theta179
    initial 1048576
    fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta180
    hyperid 29280
    name theta180
    short.name theta180
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta181
    hyperid 29281
    name theta181
    short.name theta181
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta182
    hyperid 29282
    name theta182
    short.name theta182
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta183
    hyperid 29283
    name theta183
    short.name theta183
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta184
  hyperid 29284
  name theta184
  short.name theta184
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta185
  hyperid 29285
  name theta185
  short.name theta185
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta186
  hyperid 29286
  name theta186
  short.name theta186
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta187
  hyperid 29287
  name theta187
  short.name theta187
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta188
  hyperid 29288
  name theta188
  short.name theta188

```

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta189
    hyperid 29289
    name theta189
    short.name theta189
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta190
    hyperid 29290
    name theta190
    short.name theta190
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta191
    hyperid 29291
    name theta191
    short.name theta191
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta192
    hyperid 29292
    name theta192
    short.name theta192
    initial 1048576
    fixed FALSE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta193
  hyperid 29293
  name theta193
  short.name theta193
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta194
  hyperid 29294
  name theta194
  short.name theta194
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta195
  hyperid 29295
  name theta195
  short.name theta195
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta196
  hyperid 29296
  name theta196
  short.name theta196
  initial 1048576
  fixed FALSE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta197
  hyperid 29297

```

```

name theta197
short.name theta197
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta198
hyperid 29298
name theta198
short.name theta198
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta199
hyperid 29299
name theta199
short.name theta199
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta200
hyperid 29300
name theta200
short.name theta200
initial 1048576
fixed FALSE
prior none
param
to.theta function(x) x
from.theta function(x) x
theta201
hyperid 29301
name theta201
short.name theta201
initial 1048576
fixed FALSE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta202
    hyperid 29302
    name theta202
    short.name theta202
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta203
    hyperid 29303
    name theta203
    short.name theta203
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta204
    hyperid 29304
    name theta204
    short.name theta204
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta205
    hyperid 29305
    name theta205
    short.name theta205
    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```



#### theta206

hyperid 29306  
name theta206  
short.name theta206  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta207

hyperid 29307  
name theta207  
short.name theta207  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta208

hyperid 29308  
name theta208  
short.name theta208  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta209

hyperid 29309  
name theta209  
short.name theta209  
initial 1048576  
fixed FALSE  
prior none  
param  
to.theta function(x) x  
from.theta function(x) x

#### theta210

hyperid 29310  
name theta210  
short.name theta210

```

    initial 1048576
    fixed FALSE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

n.div.by -1

n.required TRUE

set.default.values TRUE

pdf iidkd

```

## Example

Just simulate some data and estimate the parameters back. This is for `order=4`.

```

library(mvtnorm)

n <- 300
m <- 4
N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
  for (j in 1:m) {
    if (i != j) {
      Sigma[i, j] <- rho^abs(i-j) * sqrt(Sigma[i, i] * Sigma[j, j])
    }
  }
}

y <- c()
yy <- rmvnorm(n, sigma = Sigma)
for(i in 1:m) {
  y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
  ## set parameters using 'theta1'.
  ## these are the default parameters.
  hyper = list(theta1 = list(
    param = c(100, rep(1, m), rep(0, m*(m-1)/2))))),

```

```

data = data.frame(i = 1:N, y),
## fix precision as we have exact observations
control.family = list(hyper = list(
                                prec = list(initial = 15, fixed = TRUE))),
verbose = FALSE)

## this is how the internal parameters are defined
L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))

## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
                           mle = c(diag(LL), LL[lower.tri(LL)]),
                           inla = r$mode$theta))

## this gives a list of sampled matrices (stdev's and correlations)
xx <- inla.iidkd.sample(10^4, r, "i")
## compute the mean
qq <- matrix(rowMeans(matrix(unlist(xx), nrow = m^2)), m, m)

iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(diag(qq), qq[lower.tri(qq)]),
                           true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))

```

## Notes

Note that for higher values of  $k$  the only integration scheme that is supported is `control.inla=list(int.strategy="eb")`.