

# New 0inflated models: Poisson & Binomial

## Parametrisation

This is a new implementation (Nov'22) of zero-inflated Poisson and Binomial likelihood, where we will allow for a linear predictor in both the zero-inflation and in the mean, but one of them needs to consist of fixed effects only. This means the setup will be somewhat different than for other likelihood models.

## Details

The zero-inflated likelihood  $f_0(y|\dots)$  is defined as

$$f_0(y|\eta_1, \eta_2) = p(\eta_1)1_{[y=0]} + (1 - p(\eta_1))f(y|\eta_2)$$

where  $f(y|\dots)$  is either Poisson or Binomial. We allow for two linear predictors in the model, but one needs to be “simple” (i.e. only consists of fixed effects). The other is general and defined via the formula. Normally, the zero-inflation probability is simpler (`family="0..."`)

$$\eta_1 = \text{simple} \quad \eta_2 = \text{formula}$$

but they can also be swapped (`family="0...S"`)

$$\eta_1 = \text{formula} \quad \eta_2 = \text{simple}$$

## Link-function

This is similar to Poisson and Binomial.

The link-function for the 'simple'-model must be given by argument `link.simple` in the `control.family`-argument. Only link-models without covariates/parameters are currently available. The examples later on show how this is done.

## Hyperparameters

All parameters in the simple model are treated as hyperparameters. The  $j$ 'th element of  $\eta_1$  is

$$(\eta_1)_j = \sum_{i=1}^m m\beta_i x_{ij}$$

for covariates  $x_1, \dots$ , where  $m$  is maximum 10. An intercept in this model has to be defined manually by adding a constant covariate vector.

## Specification

- `family="0poisson"`
- `family="0poissonS"`
- `family="0binomial"`
- `family="0binomialS"`
- Required arguments: As for the Poisson and Binomial (but how these arguments are given, will differ). Optional argument `link.simple`.

## Hyperparameter specification and default values

### Opoisson

doc New 0-inflated Poisson

#### hyper

##### theta1

hyperid 56201  
name beta1  
short.name beta1  
output.name beta1 for Opoisson observations  
output.name.intern beta1 for Opoisson observations  
initial -4  
fixed FALSE  
prior normal  
param -4 10  
to.theta function(x) x  
from.theta function(x) x

##### theta2

hyperid 56202  
name beta2  
short.name beta2  
output.name beta2 for Opoisson observations  
output.name.intern beta2 for Opoisson observations  
initial 0  
fixed FALSE  
prior normal  
param 0 10  
to.theta function(x) x  
from.theta function(x) x

##### theta3

hyperid 56203  
name beta3  
short.name beta3  
output.name beta3 for Opoisson observations  
output.name.intern beta3 for Opoisson observations  
initial 0  
fixed FALSE  
prior normal  
param 0 10  
to.theta function(x) x  
from.theta function(x) x

##### theta4

hyperid 56204  
name beta4  
short.name beta4  
output.name beta4 for Opoisson observations  
output.name.intern beta4 for Opoisson observations  
initial 0  
fixed FALSE

```

    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 56205
    name beta5
    short.name beta5
    output.name beta5 for Opoisson observations
    output.name.intern beta5 for Opoisson observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56206
    name beta6
    short.name beta6
    output.name beta6 for Opoisson observations
    output.name.intern beta6 for Opoisson observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56207
    name beta7
    short.name beta7
    output.name beta7 for Opoisson observations
    output.name.intern beta7 for Opoisson observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56208
    name beta8
    short.name beta8
    output.name beta8 for Opoisson observations
    output.name.intern beta8 for Opoisson observations
    initial 0
    fixed FALSE
    prior normal

```

```

    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56209
    name beta9
    short.name beta9
    output.name beta9 for Opoisson observations
    output.name.intern beta9 for Opoisson observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 56210
    name beta10
    short.name beta10
    output.name beta10 for Opoisson observations
    output.name.intern beta10 for Opoisson observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x

status experimental
survival FALSE
discrete TRUE
link default log quantile
link.simple default logit cauchit probit cloglog ccloglog
pdf 0inflated

OpoissonS
doc New 0-inflated Poisson Swap
hyper
    theta1
        hyperid 56301
        name beta1
        short.name beta1
        output.name beta1 for OpoissonS observations
        output.name.intern beta1 for OpoissonS observations
        initial -4
        fixed FALSE
        prior normal
        param -4 10

```

```

    to.theta function(x) x
    from.theta function(x) x
theta2
  hyperid 56302
  name beta2
  short.name beta2
  output.name beta2 for OpoissonS observations
  output.name.intern beta2 for OpoissonS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta3
  hyperid 56303
  name beta3
  short.name beta3
  output.name beta3 for OpoissonS observations
  output.name.intern beta3 for OpoissonS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta4
  hyperid 56304
  name beta4
  short.name beta4
  output.name beta4 for OpoissonS observations
  output.name.intern beta4 for OpoissonS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta5
  hyperid 56305
  name beta5
  short.name beta5
  output.name beta5 for OpoissonS observations
  output.name.intern beta5 for OpoissonS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x

```

```

    from.theta function(x) x
theta6
    hyperid 56306
    name beta6
    short.name beta6
    output.name beta6 for OpoissonS observations
    output.name.intern beta6 for OpoissonS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56307
    name beta7
    short.name beta7
    output.name beta7 for OpoissonS observations
    output.name.intern beta7 for OpoissonS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56308
    name beta8
    short.name beta8
    output.name beta8 for OpoissonS observations
    output.name.intern beta8 for OpoissonS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56309
    name beta9
    short.name beta9
    output.name beta9 for OpoissonS observations
    output.name.intern beta9 for OpoissonS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x

```

#### theta10

hyperid 56310  
name beta10  
short.name beta10  
output.name beta10 for OpoissonS observations  
output.name.intern beta10 for OpoissonS observations  
initial 0  
fixed FALSE  
prior normal  
param 0 10  
to.theta function(x) x  
from.theta function(x) x

status experimental

survival FALSE

discrete TRUE

link default logit loga cauchit probit cloglog ccloglog loglog log sslogit logitoffset quantile pquantile

link.simple default log

pdf 0inflated

#### Obinomial

doc New 0-inflated Binomial

#### hyper

##### theta1

hyperid 56401  
name beta1  
short.name beta1  
output.name beta1 for Obinomial observations  
output.name.intern beta1 for Obinomial observations  
initial -4  
fixed FALSE  
prior normal  
param -4 10  
to.theta function(x) x  
from.theta function(x) x

##### theta2

hyperid 56402  
name beta2  
short.name beta2  
output.name beta2 for Obinomial observations  
output.name.intern beta2 for Obinomial observations  
initial 0  
fixed FALSE  
prior normal  
param 0 10  
to.theta function(x) x  
from.theta function(x) x

##### theta3

```

hyperid 56403
name beta3
short.name beta3
output.name beta3 for 0binomial observations
output.name.intern beta3 for 0binomial observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta4
hyperid 56404
name beta4
short.name beta4
output.name beta4 for 0binomial observations
output.name.intern beta4 for 0binomial observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta5
hyperid 56405
name beta5
short.name beta5
output.name beta5 for 0binomial observations
output.name.intern beta5 for 0binomial observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta6
hyperid 56406
name beta6
short.name beta6
output.name beta6 for 0binomial observations
output.name.intern beta6 for 0binomial observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta7
hyperid 56407

```



```

name beta7
short.name beta7
output.name beta7 for 0binomial observations
output.name.intern beta7 for 0binomial observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta8
  hyperid 56408
  name beta8
  short.name beta8
  output.name beta8 for 0binomial observations
  output.name.intern beta8 for 0binomial observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta9
  hyperid 56409
  name beta9
  short.name beta9
  output.name beta9 for 0binomial observations
  output.name.intern beta9 for 0binomial observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 56410
  name beta10
  short.name beta10
  output.name beta10 for 0binomial observations
  output.name.intern beta10 for 0binomial observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
status experimental
survival FALSE

```

discrete TRUE  
link default logit loga cauchit probit cloglog ccloglog loglog log  
link.simple default logit cauchit probit cloglog ccloglog  
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## Opoisson

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hyper

### theta1

hyperid 56501  
name beta1  
short.name beta1  
output.name beta1 for 0binomialS observations  
output.name.intern beta1 for 0binomialS observations  
initial -4  
fixed FALSE  
prior normal  
param -4 10  
to.theta function(x) x  
from.theta function(x) x

### theta2

hyperid 56502  
name beta2  
short.name beta2  
output.name beta2 for 0binomialS observations  
output.name.intern beta2 for 0binomialS observations  
initial 0  
fixed FALSE  
prior normal  
param 0 10  
to.theta function(x) x  
from.theta function(x) x

### theta3

hyperid 56503  
name beta3  
short.name beta3  
output.name beta3 for 0binomialS observations  
output.name.intern beta3 for 0binomialS observations  
initial 0  
fixed FALSE  
prior normal  
param 0 10  
to.theta function(x) x  
from.theta function(x) x

### theta4

hyperid 56504  
name beta4

```

short.name beta4
output.name beta4 for 0binomialS observations
output.name.intern beta4 for 0binomialS observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta5
  hyperid 56505
  name beta5
  short.name beta5
  output.name beta5 for 0binomialS observations
  output.name.intern beta5 for 0binomialS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta6
  hyperid 56506
  name beta6
  short.name beta6
  output.name beta6 for 0binomialS observations
  output.name.intern beta6 for 0binomialS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta7
  hyperid 56507
  name beta7
  short.name beta7
  output.name beta7 for 0binomialS observations
  output.name.intern beta7 for 0binomialS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta8
  hyperid 56508
  name beta8
  short.name beta8

```

```

output.name beta8 for 0binomialS observations
output.name.intern beta8 for 0binomialS observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x
theta9
  hyperid 56509
  name beta9
  short.name beta9
  output.name beta9 for 0binomialS observations
  output.name.intern beta9 for 0binomialS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 56510
  name beta10
  short.name beta10
  output.name beta10 for 0binomialS observations
  output.name.intern beta10 for 0binomialS observations
  initial 0
  fixed FALSE
  prior normal
  param 0 10
  to.theta function(x) x
  from.theta function(x) x
status experimental
survival FALSE
discrete TRUE
link default logit loga cauchit probit cloglog ccloglog loglog log
link.simple default logit cauchit probit cloglog ccloglog
pdf 0inflated

```

## Example: Poisson

```
sim.poisson <- function(prob, m)
{
  stopifnot(length(prob) == length(m) && length(prob) > 0)
  n <- length(m)
  y <- numeric(n)
  event <- (runif(n) < prob)
  idx.zero <- which(event)
  idx.non.zero <- which(!event)
  y[idx.zero] <- 0
  y[idx.non.zero] <- rpois(length(idx.non.zero), lambda = m[idx.non.zero])
  return (y)
}

## chose link-function to use for the zero-inflation probability
link.simple <- "logit"
inv.link <- inla.link.invlogit
## link.simple <- "probit"
## inv.link <- inla.link.invprobit
## link.simple <- "cloglog"
## inv.link <- inla.link.invcloglog

n <- 1000
z <- rnorm(n, sd = 0.3)
x <- rnorm(n, sd = 0.2)
xx <- rnorm(n, sd = 0.3)
zz <- rnorm(n, sd = 0.2)
E <- runif(n, min = 0.8, max = 1/0.8)

beta <- c(1, 1.1, 2.1, 0, -2, 1.2, 2.2, 0)
eta2 <- beta[1] + beta[2] * xx + beta[3] * zz + beta[4] * xx * zz
eta1 <- beta[5] + beta[6] * x + beta[7] * z + beta[8] * x * z
prob <- inv.link(eta1)
m <- E*exp(eta2)

ok <- FALSE
while(!ok) {
  y <- sim.poisson(prob, m)
  ok <- !all(y == 0)
}

## head(data.frame(y, E, x, z, xx, zz))

r <- inla(
  inla.mdata(cbind(y, E), cbind(1, x, z, x*z)) ~ 1 + xx + zz + xx*zz,
  family = "0poisson",
  data = data.frame(y, E, x, z, xx, zz),
  control.fixed = list(prec = 1, prec.intercept = 1),
  control.compute = list(cpo = TRUE),
  control.family = list(link.simple = link.simple,
    hyper = list(beta1 = list(param = c(0, 1)),
      beta2 = list(param = c(0, 1)),
      beta3 = list(param = c(0, 1)),
      beta4 = list(param = c(0, 1)),
      beta5 = list(param = c(0, 1))))))

rr <- inla(
```

```

inla.mdata(cbind(y, E), cbind(1, xx, zz, xx*zz)) ~ 1 + x + z + x*z,
family = "0poissonS",
data = data.frame(y, E, x, z, xx, zz),
control.fixed = list(prec = 1, prec.intercept = 1),
control.compute = list(cpo = TRUE),
## in this case we need to define link.simple as the main link
control.family = list(control.link = list(model = link.simple),
                      hyper = list(beta1 = list(param = c(0, 1)),
                                   beta2 = list(param = c(0, 1)),
                                   beta3 = list(param = c(0, 1)),
                                   beta4 = list(param = c(0, 1)),
                                   beta5 = list(param = c(0, 1)))))

summary(r)
summary(rr)

res <- cbind("beta" = beta,
            "0poisson" = c(r$summary.fixed$mean, r$summary.hyperpar$mean),
            "0poissonS" = c(rr$summary.hyperpar$mean, rr$summary.fixed$mean))
res <- cbind(res,
            diff = (res[, 2]-beta),
            diffS = (res[, 3]-beta),
            "diff/sd" = (res[, 2]-beta) / c(r$summary.fixed$sd, r$summary.hyperpar$sd),
            "diffS/sd" = (res[, 3]-beta) / c(rr$summary.hyperpar$sd, rr$summary.fixed$sd))
mm <- nrow(res) %/% 2
rownames(res) <- c(paste0("beta", 1:mm, ".poisson"), paste0("beta", 1:mm, ".prob"))
print(round(dig = 2, res))

```

## Example: Binomial

```
sim.binomial <- function(prob, p, size)
{
  ## - prob=zero-inflation-prob
  ## - binomial(size, p)
  stopifnot(length(prob) == length(p) && length(prob) == length(size)
            && length(prob) > 0)
  n <- length(prob)
  y <- numeric(n)
  event <- (runif(n) < prob)
  idx.zero <- which(event)
  idx.non.zero <- which(!event)
  y[idx.zero] <- 0
  y[idx.non.zero] <- rbinom(length(idx.non.zero),
                           size = size[idx.non.zero],
                           prob = p[idx.non.zero])

  return (y)
}

n <- 1000
z <- rnorm(n, sd = 0.3)
x <- rnorm(n, sd = 0.2)
xx <- rnorm(n, sd = 0.3)
zz <- rnorm(n, sd = 0.2)
Ntrials <- sample(1:10, n, replace = TRUE)

## chose link-function to use for the zero-inflation probability
link.simple <- "logit"
inv.link <- inla.link.invlogit
## link.simple <- "probit"
## inv.link <- inla.link.invprobit
## link.simple <- "cloglog"
## inv.link <- inla.link.invcloglog

beta <- c(1, 1.1, 2.1, 0, -2, 1.2, 2.2, 0)
eta2 <- beta[1] + beta[2] * xx + beta[3] * zz + beta[4] * xx * zz
eta1 <- beta[5] + beta[6] * x + beta[7] * z + beta[8] * x * z
prob <- inv.link(eta1)
p <- 1/(1 + exp(-eta2))

ok <- FALSE
while(!ok) {
  y <- sim.binomial(prob, p, Ntrials)
  ok <- !all(y == 0)
}

## head(data.frame(y, Ntrials, x, z, xx, zz))

r <- inla(
  inla.mdata(cbind(y, Ntrials), cbind(1, x, z, x*z)) ~ 1 + xx + zz + xx*zz,
  family = "0binomial",
  data = data.frame(y, Ntrials, x, z, xx, zz),
  control.fixed = list(prec = 1, prec.intercept = 1),
  control.compute = list(cpo = TRUE),
  control.family = list(link.simple = link.simple,
                        hyper = list(beta1 = list(param = c(0, 1)),
                                      beta2 = list(param = c(0, 1)),
```

```

        beta3 = list(param = c(0, 1)),
        beta4 = list(param = c(0, 1)),
        beta5 = list(param = c(0, 1))))

rr <- inla(
  inla.mdata(cbind(y, Ntrials), cbind(1, xx, zz, xx*zz)) ~ 1 + x + z + x*z,
  family = "0binomialS",
  data = data.frame(y, Ntrials, x, z, xx, zz),
  control.fixed = list(prec = 1, prec.intercept = 1),
  control.compute = list(cpo = TRUE),
  ## in this case we need to define link.simple as the main link
  control.family = list(control.link = list(model = link.simple),
    hyper = list(beta1 = list(param = c(0, 1)),
      beta2 = list(param = c(0, 1)),
      beta3 = list(param = c(0, 1)),
      beta4 = list(param = c(0, 1)),
      beta5 = list(param = c(0, 1)))))

summary(r)
summary(rr)

res <- cbind("beta" = beta,
  "0binomial" = c(r$summary.fixed$mean, r$summary.hyperpar$mean),
  "0binomialS" = c(rr$summary.hyperpar$mean, rr$summary.fixed$mean))
res <- cbind(res,
  diff = (res[, 2]-beta),
  diffS = (res[, 3]-beta),
  "diff/sd" = (res[, 2]-beta) / c(r$summary.fixed$sd, r$summary.hyperpar$sd),
  "diffS/sd" = (res[, 3]-beta) / c(rr$summary.hyperpar$sd, rr$summary.fixed$sd))
mm <- nrow(res) %/% 2
rownames(res) <- c(paste0("beta", 1:mm, ".binomial"), paste0("beta", 1:mm, ".prob"))
print(round(dig = 2, res))

```