

# The Kumaraswamy distribution

## Parametrisation

The Kumaraswamy distribution is

$$f(y) = \alpha\beta y^{\alpha-1}(1-y^\alpha)^{\beta-1}$$

for  $0 < y < 1$  and  $\alpha, \beta > 0$ . The cumulative distribution function is

$$F(y) = 1 - (1 - y^\alpha)^\beta.$$

The parametrisation is given in terms of the quantile function

$$\kappa(q) = \left(1 - (1 - q)^{1/\beta}\right)^{1/\alpha}$$

and the precision parameter  $\phi$ ,

$$\phi(q) = -\ln\left(1 - (1 - q)^{1/\beta}\right)$$

for *fixed* value of  $0 < q < 1$ .

## Link-function

The quantile  $\kappa$  to the linear predictor by

$$\text{logit}(\kappa) = \eta$$

using the default logit link-function.

## Hyperparameters

The hyperparameter is

$$\phi = \exp(S\theta)$$

and the prior is defined on  $\theta$ . The constant  $S$  currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of  $\alpha$  can make a huge difference.

## Specification

- `family="qkumar"`
- Required arguments:  $y$  and the quantile  $q$ .

The quantile is given as `control.family=list(control.link = list(quantile=q))`.

## Hyperparameter spesification and default values

`doc` A quantile version of the Kumar likelihood

`hyper`

`theta`

`hyperid` 60001

`name` precision parameter

`short.name` prec

`output.name` precision for qkumar observations

```

output.name.intern log precision for qkumar observations
initial 1
fixed FALSE
prior loggamma
param 1 0.1
to.theta function(x, sc = 0.1) log(x) / sc
from.theta function(x, sc = 0.1) exp(sc * x)

survival FALSE

discrete FALSE

link default logit loga cauchit

pdf qkumar

```

## Example

```

rkumar = function(n, eta, phi, q=0.5)
{
  kappa = eta
  beta = log(1-q)/log(1-exp(-phi))
  alpha = log(1- (1-q)^(1/beta)) / log(kappa)
  u = runif(n)
  y = (1-u^(1/beta))^(1/alpha)
  return (y)
}

n = 100
q = 0.5
phi = 1
x = rnorm(n, sd = 1)
eta = inla.link.invlogit(1 + x)
y = rkumar(n, eta, phi, q)
r = inla(y ~ 1 + x,
  data = data.frame(y, x),
  family = "qkumar",
  control.family = list(control.link=list(quantile = q)))
summary(r)

```

## Notes

None.