

# The Gammacount-likelihood

## Parametrisation

The Gammacount-distribution is a discrete probability distribution on  $0, 1, 2, 3, \dots$ , where

$$\text{Prob}(y) = G(y\alpha, \beta) - G((y+1)\alpha, \beta)$$

where

$$G(a, b) = \frac{1}{\Gamma(a)} \int_0^b x^{a-1} \exp(-x) dx.$$

The reciprocal of the expected waiting time depends on the linear predictor

$$(\alpha/\beta)^{-1} = E \exp(\eta),$$

so that high values of  $\eta$  corresponds to high values of  $y$ , and low values of  $\eta$  corresponds to low values of  $y$ , and  $\log(E)$  is an offset. In the case where  $\alpha = 1$ , then we are back to the Poisson.

## Link-function

The linear predictor  $\eta$  is linked to the reciprocal of the expected waiting time, by a log-link,

$$(\alpha/\beta)^{-1} = E \exp(\eta), \quad E > 0.$$

## Hyperparameter

The hyperparameter is the parameter  $\alpha$ , which is represented as

$$\alpha = \exp(\theta)$$

and the prior is defined on  $\theta$ .

## Specification

- family="gammacount"
- Required arguments:  $y$  (and  $E$ , default one)

## Hyperparameter spesification and default values

doc A Gamma generalisation of the Poisson likelihood

hyper

theta

hyperid 59001

name log alpha

short.name alpha

output.name Log-alpha parameter for Gammacount observations

output.name.intern Alpha parameter for Gammacount observations

initial 0

fixed FALSE

prior pc.gammacount

param 3

```

to.theta function(x) log(x)
from.theta function(x) exp(x)

```

```
survival FALSE
```

```
discrete FALSE
```

```
link default log
```

```
pdf gammacount
```

## Example

In the following example we estimate the parameters in a simulated example.

```

G = function(Alpha, Beta) {
  return (pgamma(Beta, shape=Alpha, rate=1))
}

n = 1000
x = rnorm(n)
eta = 1 + x
alpha = 1.5
T = 1
m = 100
y = numeric(n)
prob = numeric(m+1)

for(i in 1:n) {

  ## compute the discrete probability distribution and
  ## then sample from it
  for(j in 1:m) {
    yy = j-1
    beta = alpha * exp(eta[i])
    prob[j] = (G(yy*alpha, beta*T) -
               G((yy+1)*alpha, beta*T))
  }
  y[i] = sample(0:m, size=1, prob = prob)
}

r = (inla(y ~ 1 + x,
          data = data.frame(y, x),
          family = "gammacount"))
summary(r)

```

## Notes

None.