The RW2d-model

Parametrization

The 2-dimensional random walk model is defined on a regular grid. The full conditional distributions for the nodes in the interior of the grid are given by:

$$
\begin{aligned}
\mathrm{E}(x_i \mid \mathbf{x}_{-i}, \tau) &= \frac{1}{20}\left(8 \begin{matrix} \circ & \circ & \circ & \circ & \circ \\ \circ & \circ & \bullet & \circ & \circ \\ \circ & \bullet & \circ & \bullet & \circ \\ \circ & \circ & \bullet & \circ & \circ \\ \circ & \circ & \circ & \circ & \circ \end{matrix} \; - 2 \begin{matrix} \circ & \circ & \circ & \circ & \circ \\ \circ & \bullet & \circ & \bullet & \circ \\ \circ & \circ & \circ & \circ & \circ \\ \circ & \bullet & \circ & \bullet & \circ \\ \circ & \circ & \circ & \circ & \circ \end{matrix} \; - 1 \begin{matrix} \circ & \circ & \bullet & \circ & \circ \\ \circ & \circ & \circ & \circ & \circ \\ \bullet & \circ & \circ & \circ & \bullet \\ \circ & \circ & \circ & \circ & \circ \\ \circ & \circ & \bullet & \circ & \circ \end{matrix} \right), \qquad (1) \\
\mathrm{Prec}(x_i \mid \mathbf{x}_{-i}, \tau) &= 20\tau. \qquad (2)
\end{aligned}
$$

Necessary corrections to to equations (1) and (2) near the boundary can be found using the stencils in [Terzopoulos, 1988]. For a detailed description of this model see [Rue and Held, 2005, Sec. 3.4.2].

Hyperparameters

The precision parameter τ is the only hyperparameter, $\theta = \tau$. It is represented internally as $\log\tau$ and the prior is assigned to $\log\tau$.

Specification

The `rw2d` model is specified insiede the `f()` function as:

```
f(<whatever>,model="rw2d",
  nrow=<n.of rows>,ncol=<n.of columns>, bvalue=<bvalue>,
  hyper= <hyper>, scale.model = FALSE)
```

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

The argument `bvalue` defines how the boundary is handled. If `bvalue=0` (default) then precision matrix is created like we are condition on 0's outside the region. This model is proper. If `bvalue=1` then only points inside the region is considered, and non-standard stencils are used near the boundary. This model is improper.

Hyperparameter spesification and default values

doc `Thin-plate spline model`

hyper

theta

- **hyperid** `32001`
- **name** `log precision`
- **short.name** `prec`
- **initial** `4`
- **fixed** `FALSE`
- **prior** `loggamma`
- **param** `1 5e-05`
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

constr `TRUE`

nrow.ncol TRUE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values TRUE

pdf rw2d

Example

```
nrow=50
ncol=25
n = nrow*ncol
s.mat=matrix(NA,nrow=nrow,ncol=ncol)
j=1:ncol
for(i in 1:nrow)
    s.mat[i,j] = 0.1*(i+2*j)

## a covariate
z.mat=matrix(runif(nrow*ncol),nrow,ncol)

## noise
noise.mat=matrix(rnorm(nrow*ncol, sd=0.3),nrow,ncol)

## make simulated data
y.mat = s.mat  + 0.5*z.mat + noise.mat

## convert matrices to the internal representation in INLA
y = inla.matrix2vector(y.mat)
z = inla.matrix2vector(z.mat)
node = 1:n
formula= y ~ z + f(node, model="rw2d", nrow=nrow, ncol=ncol)
data=data.frame(y=y,z=z,node=node)
## fit the model
result=inla(formula, family="gaussian", data=data)

#plot the posterior mean for ‘node’ with the truth
dev.new()
INLA:::inla.display.matrix(s.mat)
dev.new()
INLA:::inla.display.matrix(INLA:::inla.vector2matrix(result$summary.random$node$mean,nrow,ncol
```

Notes

All indexes in the R-INLA library are one-dimensional so an appropriate mapping is required to get it into the ordering defined internally in `inla`; see `?inla.matrix2vector`, `?inla.vector2matrix`, `?inla.node2lattice` and `?inla.lattice2node`.

The $\frac{n-r}{2}\log(|R|^*)$-part (with $r = 3$) of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here, $Q = \tau R$.

References

[Rue and Held, 2005] Rue, H. and Held, L. (2005). *Gaussian Markov Random Fields: Theory and Applications*, volume 104 of *Monographs on Statistics and Applied Probability.* Chapman & Hall, London.

[Terzopoulos, 1988] Terzopoulos, D. (1988). The computation of visible-surface representations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 10(4):417–438.