!!!!NOT USED ANYMORE!!!

The Wishart model for correlated effects

This model is availabe for dimensions $p = 2, 3$, we describe in detail the case for $p = 2$ and the the case for $p = 3$.

Parametrization

The 2-dimesional Wishard model is used if one want to define the model for the linear predictor η as:

$$\eta = a + b$$

where a and b are correlated

$$\begin{pmatrix} a \\ b \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)$$

with covariance matrix $\mathbf{W}^{-1}$

$$\mathbf{W}^{-1} = \begin{pmatrix} 1/\tau_a & \rho\sqrt{\tau_a\tau_b} \\ \rho\sqrt{\tau_a\tau_b} & 1/\tau_b \end{pmatrix} \tag{1}$$

and τ_a, τ_b and ρ are the hyperparameters. In this case the following model is implemented for the precision matrix $\mathbf{W}$

$$\mathbf{W} \;\sim\; \text{Wishart}_p(r, \mathbf{R}^{-1}), \quad p = 2$$

where the Wishart distribution has density

$$\pi(\mathbf{W}) = c^{-1}|\mathbf{W}|^{(r-(p+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > p + 1$$

and

$$c = 2^{(rp)/2}|\mathbf{R}|^{-r/2}\pi^{(p(p-1))/4} \prod_{j=1}^{p} \Gamma((r + 1 - j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (p + 1)).$$

Hyperparameters

The hyperparameters are

$$\theta = (\log\tau_a, \log\tau_b, \tilde{\rho})$$

where

$$\rho = 2\frac{\exp(\tilde{\rho})}{\exp(\tilde{\rho}) + 1} - 1$$

The prior-parameters are

$$(r, R_{11}, R_{22}, R_{12})$$

where

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

and $r_{12} = R_{21}$ due to symmetry.

The inla function reports posterior distribution for the hyperparameters τ_a, τ_b, ρ in equation (1).

The prior for θ is **fixed** to be wishart

Specification

The model 2d wishart for

$$\eta = a + b$$

is specified as

```
y~f(a,model="2diidwishartp1",param=<param.vector(4 elements)>)
  +f(b,model="2diidwishartp2")
```

The parameters for the Wishart distribution are specified *only* for 2diidwishartp1

Example

In this example we implement the model

$$y|\eta \sim \text{Pois}(\exp(\eta))$$

where

$$\eta = a + b + c$$

and b and c are correlated as described above.

```
n=100
#set hyperparameters
r=4
R11=1
R22=2
R12=0.1
R=matrix(c(R11,R12,R12,R22),2,2)
S=solve(R)
#these are needed to simulate from a wishart prior
# and sample from a multivariate normal
library(MCMCpack)
library(mvtnorm)
W=rwish(r, S)
cc=rmvnorm(n, mean=c(0,0), sigma=solve(W))
a=cc[,1]
b=cc[,2]
#simulate data
x1=1:n
x2=1:n
eta=0.1+a+b
y=rpois(n,exp(eta))
data=data.frame(y=y,x1=1:n,x2=1:n)
#fit the model
```

```
formula=y~f(x1,model="2diidwishartp1",param=c(4,1,2,0.1))+f(x2,model="2diidwishartp2")
result=inla(formula,family="poisson",data=data)
```

Notes

If more than one pair of `2diidwishartp1/2` is defined, the following rule is used to determine the match between `p(art)1` and `p(art)2`.

> The first occurrence of `2diidwishartp1` belongs with the first occurrence of `2diidwishartp2`. The second occurrence of `2diidwishartp1` belongs with the second occurrence of `2diidwishartp2` and so on.

Three dimensional case

The previous formulation is also available for 3D. In this case the hyperparameters are

$$\theta = (\log \tau_1, \log \tau_2, \log \tau_3, \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23})$$

In this case the name the prior is **fixed** to be `Wishart3d`. The parameters in the prior are

$$parameters = \; r \; R_{11} \; R_{22} \; R_{33} \; R_{12} \; R_{13} \; R_{23}$$

where

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{23} \\ R_{13} & R_{23} & R_{33} \end{bmatrix}$$

The reported hyperparameters are the marginal precisions τ_1, τ_2 and τ_3 and the correlations ρ_{12}, ρ_{13} and ρ_{23}. The model names are as given in the following example.

```
formula2 <- Y ~ f(diid.part0,model="3diidwishartp1",
                  param=c(7,1,2,3,0.1,0.2,0.3))
                f(diid.part1,model="3diidwishartp2") +
                f(diid.part2,model="3diidwishartp3") +
```

If more than one pair of `3diidwishartp1/2/3` is defined, the following rule is used to determine the match between *p(art)1, p(art)2* and *p(art)3*.

> The first occurrence of `3diidwishartp1` belongs with the first occurrence of `3diidwishartp2` and `3diidwishartp3`. The second occurrence of `3diidwishartp1` belongs with the second occurrence of `3diidwishartp2` and the second occurrence of `3diidwishartp3` , and so on.