

# Cox Proportional Hazards Model

## Parametrisation

In the Cox proportional hazards model, defines the hazard rate as:

$$h(t) = h_0(t) \exp(\eta)$$

where

$h_0(\cdot)$ : baseline hazard

$\eta$ : linear predictor

We start from a finite partition of the time axis  $0 = s_0 < s_1 < \dots, s_K$  and assume the baseline hazard to be constant in each time interval

$$h_0(t) = \exp(b_k) \text{ for } t \in (s_{k-1}, s_k], \quad k = 1, \dots, K$$

and assign  $\mathbf{b} = (b_1, \dots, b_K)$  a Gaussian prior (RW1 or RW2) with unknown precision  $\tau_b$

## Link-function

The parameter  $\eta$  is the linear predictor

## Hyperparameters

The log precision  $\log \tau_b$  for the piecewise constant hazard

## Specification

- `family="coxph"`
- Required arguments:
  - `y` (to be given in a format by using `inla.surv()` function )
  - `control.hazard = list()` to control the prior for the piecewise constant hazard, see `?control.hazard` for more information.

## Hyperparameter specification and default values

### The “RW1” model for the hazard

`doc` A random walk of order 1 for the log-hazard

`hyper`

`theta`

```
hyperid 54001
name log precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

## The “RW2” model for the hazard

doc A random walk of order 2 for the log-hazard

hyper

```
theta
  hyperid 55001
  name log precision
  short.name prec
  initial 4
  fixed FALSE
  prior loggamma
  param 1 5e-05
  to.theta function(x) log(x)
  from.theta function(x) exp(x)
```

## Example

In the following example we estimate the baseline hazard in a simulated case

```
n = 1000
alpha = 2
beta = 2
x = runif(n)
eta = 1+beta*x
lambda = exp(eta)
y = rweibull(n, shape= alpha, scale= lambda^(1/-alpha))
event = rep(1,n)
data = list(y=y, event=event, x=x)

formula=inla.surv(y,event)~ x

model=inla(formula, family = "coxph", data=data, verbose=T,
  control.hazard=list(model="rw1", n.intervals=20))
```

## Notes

- The Cox model can be used only for uncensored or right censored data.
- The model for the piecewise constant baseline hazard is specified through `control.hazard`
- A general frame work to represent time is given by `inla.surv`
- If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routines giving error messages like

```
file: smtp-taucs.c  hgid: 891deb69ae0c  date: Tue Nov 09 22:34:28 2010 +0100
Function: GMRFLib_build_sparse_matrix_TAUCS(), Line: 611, Thread: 0
Variable evaluates to NAN/INF. This does not make sense. Abort...
```

If you encounter this problem, try to scale the observations, `time = time / max(time)` or similar, before running `inla()`.