

## Linkmodel: powerlogit

### Parametrization

This is the link that map  $p \in (0, 1)$  into  $x \in \Re$ , where

$$F^\beta(x) = p, \quad \beta > 0$$

and  $F(x)$  is the cummulative distribution function for the logit,

$$F(x) = \frac{1}{1 + \exp(-x)}.$$

This link is renormalized so its corresponding density have zero mean and unit variance for every value of  $\beta$ .

### Hyperparameters

The parameter  $\beta$  represent the power

$$\beta = \exp(\theta_1)$$

and the prior is defined on  $\theta_1$ .

The intercept is represented by a quantile level  $\alpha$ , where

$$\alpha = \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior is defined on  $\theta_2$ .

### Specification

Use `model="powerlogit"` within `control.link`.

### Hyperparameter spesification and default values

`doc` Power logit link

`hyper`

`theta1`

```
hyperid 49131
name power
short.name power
initial 0.00123456789
fixed FALSE
prior normal
param 0 10
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

`theta2`

```
hyperid 49132
name intercept
short.name p0
```

```

initial 0
fixed FALSE
prior logitbeta
param 1 1
to.theta function(x) log(x / (1 - x))
from.theta function(x) exp(x) / (1 + exp(x))

```

pdf powerlogit

## Example

```

map <- function(x) log(x/(1-x))
imap <- function(x) 1/(1+exp(-x))

power.link <- function(power) {

  qfunc <- function(q) -log(q^(-1/power) -1)
  dfunc <- function(x) power * exp(-x) / (1+exp(-x))^(power + 1)
  pfunc <- function(x) 1/(1+exp(-x))^power

  dq <- 0.005
  qq <- unique(sort(c(0+exp(seq(-10, 0, by = dq)),
                    1-exp(seq(-10, 0, by = dq)))))
  qq <- qq[(qq < 1) & (qq > 0)]
  xx <- qfunc(qq)
  ## total probability, need to account for this in cdf/icdf so it will extrapolate no need
  ## do this in the m & s calculation
  p <- qq[length(qq)]
  dens <- dfunc(xx)
  dx <- (c(0, diff(xx)) + c(diff(xx), 0))/2
  dens <- (dens * dx) / sum(dens * dx)
  m = sum(xx*dens)
  s = sqrt(sum(xx^2*dens) - m^2)
  xx <- (xx-m)/s
  csum <- p * cumsum(dens)
  cdf <- splinefun(xx, map(csum), method = "monoH.FC")
  icdf <- splinefun(map(csum), xx, method = "monoH.FC")
  return(list(cdf = cdf, icdf = icdf, mean = m, sd = s))
}

n <- 500
size <- 25
power <- 0.25
link <- power.link(power)
x <- rnorm(n, sd = 0.5)
x <- x-mean(x)
intercept <- 1
prob <- imap(link$cdf(intercept + x))
y <- rbinom(n, size = size, prob = prob)

r <- inla(y ~ 1 + x,

```

```

data = data.frame(y, x),
family = "binomial",
Ntrials = size,
inla.mode = "experimental",
verbose = TRUE,
control.inla = list(int.strategy = "eb"),## cmin = 0),
control.predictor = list(compute = TRUE, link = 1),
control.fixed = list(prec = 1,
                      remove.names = "(Intercept)"),
control.family = list(
  ## start with the true values to demonstrate how its done
  control.link = list(
    model = "powerlogit",
    hyper = list(
      intercept = list(initial = link$cdf(intercept),
                       prior = "normal",
                       param = c(0,1)),
      power = list(initial = log(power),
                   param = c(0, 1),
                   fixed = FALSE))))),
num.threads = "4:1")

summary(r)

## this should be, about, the values of the intercept
print(round(dig = 4, c(intercept = intercept,
                      approximate.intercept.est =
                        as.numeric(link$icdf(r$mode$theta[2])))))

```

## Notes

- This link is EXPERIMENTAL, and new default priors will be added in the near future
- Setting the initial value for the hyperparameter “intercept” to infinity, will remove the intercept from the link-model.