Proper random walk model of order 2 (PRW2)

Parametrization

This is the proper version of the RW2 prior, defined for regular locations $1, 2, \ldots$,

$$x_t = 2\gamma x_{t-1} - \gamma^2 x_{t-2} + \epsilon_t, \qquad |\gamma| < 1$$

where ϵ_t is iid zero mean Gaussian noise. This process is made stationary with marginal precision κ, and the range, the 'distance' or 'time' to small correlation, is r defined (implicitely) as

$$\rho = \exp\left(-\frac{\sqrt{12}h}{r}\right)$$

where h is (typical) distance or time, 'step-size', between the knots, here $h = 1$. The correlation function of the process is found to be

$$\rho(h) = \left(1 + h\frac{1-\gamma^2}{1+\gamma^2}\right)\gamma^h, \qquad h = 0, 1, 2, \ldots$$

Similar to RW1 and RW2, this model allows for irregular locations (`loc`) and is then derived from the corresponding limiting process in continous time.

Note that r is in real scale, ie the scale of `loc`.

Hyperparameters

This model has two hyperparameters, the marginal precision κ and the range r. These are defined as

$$\theta_1 = \log(\kappa)$$
$$\theta_2 = \log(r)$$

and the priors are given for (θ_1, θ_2).

Specification

This model is given as `f(loc, model="prw2", ...)`.

Note that `constr=FALSE` by default for this model, unlike the RW2 model.

The default is use PC-priors for both parameters, see `inla.doc("pc.prec")` and `inla.doc("pc.prw2.range")`

The prior `pc.prw2.range` is currently fixed and not possible to replace, as this prior is intrinsic to the PRW2 model construction.

Hyperparameter spesification and default values

doc `Proper random walk of order 2`

hyper

theta1

hyperid `6103`
name `log precision`
short.name `prec`
prior `pc.prec`
param `1 0.01`
initial `4`

fixed FALSE

to.theta function(x) log(x)

from.theta function(x) exp(x)

theta2

hyperid 6102

name log range

short.name range

prior pc.prw2.range

param 0 0 0 0

initial 3

fixed FALSE

to.theta function(x) log(x)

from.theta function(x) exp(x)

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf prw2

Example

```
n <- 200
loc <- seq(0, n-1, length.out=n)
h.size <- diff(range(loc)) / (n-1)
f.true <- (sin(2*pi*(loc/n)^2))^3
s = 0.2
y <- f.true + rnorm(n, sd = s)

plot(loc, y, pch = 19)
lines(loc, f.true, type='l', lwd = 3, col='blue')

r <- inla(y ~ -1 + f(loc, model = "prw2", values = loc,
                     hyper = list(
                         range = list(
                             param = c(50, 0.5, h.size, 0)))),
          family = "normal",
          control.family = list(
              hyper = list(prec = list(initial = log(1/s^2),
                                       fixed = TRUE))),
          data = data.frame(y, loc))
lines(loc, r$summary.linear.predictor$mean, lwd = 3, col = "red")
```

```
## we can now change the resolution and get the 'same' results
nn <- 2*n
lloc <- seq(0, n-1, length.out = nn)
yy <- numeric(nn)
for(i in 1:n) {
    yy[1 + (i-1) * 2] <- y[i]
    yy[2 + (i-1) * 2] <- NA
}
hh.size <- diff(range(lloc)) / (nn-1)

rr <- inla(yy ~ -1 + f(lloc, model = "prw2", values = lloc,
                       hyper = list(
                           ## note that '50' is the same as its in the real scale
                           range = list(param = c(50, 0.5, hh.size, 0)))),
           family = "normal",
           control.family = list(hyper = list(prec = list(initial = log(1/s^2),
                                                          fixed = TRUE))),
           data = data.frame(yy, lloc))
lines(lloc, rr$summary.linear.predictor$mean, lwd = 3, col = "red")

## once more
nnn <- 4*n
llloc <- seq(0, n-1, length.out = nnn)
yyy <- numeric(nnn)
for(i in 1:n) {
    yyy[1 + (i-1) * 4] <- y[i]
    yyy[2:4 + (i-1) * 4] <- NA
}
hhh.size <- diff(range(llloc)) / (nnn-1)

rrr <- inla(yyy ~ -1 + f(llloc, model = "prw2", values = llloc,
                       hyper = list(
                           ## again, '50' is not changed
                           range = list(param = c(50, 0.5, hhh.size, 0)))),
           family = "normal",
           control.family = list(hyper = list(prec = list(initial = log(1/s^2),
                                                          fixed = TRUE))),
           data = data.frame(yy, lloc))
lines(llloc, rrr$summary.linear.predictor$mean, lwd = 3, col = "red")

inla.dev.new()
res <- cbind(r$summary.linear.predictor$mean,
             rr$summary.linear.predictor$mean[seq(1, nn, by = 2)],
             rrr$summary.linear.predictor$mean[seq(1, nnn, by = 4)])
pairs(res)

inla.dev.new()
plot(r$marginals.hyperpar[[2]],  pch = 19)
lines(rr$marginals.hyperpar[[2]], lwd = 3, col = "blue")
lines(rrr$marginals.hyperpar[[2]], lwd = 3, col = "red")
```

Notes

The details for this model is available in the PhD thesis with link HERE (ADD LINK LATER).